A Time-Recursive Framework for Fundamental Couplings

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Abstract

In a previous work we introduced a time–recursive dimensional hierarchy in which all physical quantities occupy exponents of a single primitive temporal unit. Building upon that framework, the present paper demonstrates that dimensionless interaction strengths—including the strong coupling α_s and the fine-structure constant α—arise naturally as geometric ratios between complementary recursion tiers. We construct a canonical dimensionless object,

$$\kappa(p, x) = p^5\, G^2\, x,$$

where p is the momentum-tier quantity $[p] = T^{-1}$, G is the gravitational coupling assigned to T^{+2}, and x is the spatial scale $[x] = T^{+1}$. Within the time–recursive hierarchy this combination satisfies

$$[p^5] + [G^2] + [x] = -5 + 4 + 1 = 0,$$

making κ a structurally enforced dimensionless constant that traces a ten-step closed walk in exponent space. This provides a direct mechanism by which interaction strengths emerge from the recursive geometry of the tier ladder, without introducing new primitives or external units.

We show that κ evaluated at appropriate momentum and coherence scales yields dimensionless values in the observed range of strong and electromagnetic couplings, and that deviations from the "bare" geometric ratio are naturally interpreted as overlap corrections between adjacent recursion tiers. These overlap terms play an analogous role to loop corrections in quantum field theory, but arise here as consequences of geometric interference within the temporal recursion lattice.

The result is a predictive framework in which all coupling constants are expressed as pure numerical ratios of recursion tiers plus small geometric overlap terms. This strengthens the claim that the time–recursive hierarchy is not merely dimensionally self-consistent, but encodes a generative mechanism underlying the observed hierarchy of interaction strengths in nature.

1 Brief Recap of the Time–Recursive Dimensional Framework

This section summarizes the elements of the original time–recursive framework developed in [1], focusing only on what is required for the construction of dimensionless coupling constants. The goal is not to re-derive the entire theory, but to provide the essential dimensional machinery on which the present paper builds.

1.1 The Global Recursion Ladder

All physical quantities are assigned a dimensional value through powers of a single primitive unit of time T:

$$[Q] = T^n.$$

The recursion rule

$$n \mapsto 2n$$

generates a sequence of "anchor" tiers

$$T^1,\ T^2,\ T^4,\ T^8,\ T^{16},\ \ldots.$$

Between these anchors sit intermediate exponents such as T^3 which allow classical kinematic identities to remain valid.

1.2 Kinematic Reconstruction

In the original framework, classical relations

$$v = \frac{L}{t}, \qquad a = \frac{L}{t^2}, \qquad p = mv$$

force the assignments

$$[t] = T^1, \qquad [L] = T^3, \qquad [v] = T^2, \qquad [a] = T^1, \qquad [m] = T^3, \qquad [p] = T^5.$$

These follow from the basic relation that spatial length must sit one step above the velocity tier, so that L can simultaneously be derived as vt and satisfy Newtonian and relativistic identities.

1.3 Interaction Laws and the Tier Structure

The weak-field gravitational law

$$g \sim \frac{GM}{R^2}$$

and the relativistic mass–energy equivalence

$$E = mc^2$$

both require

$$[G] = T^4, \qquad [g] = T^4, \qquad [E] = T^6.$$

Electromagnetism fits into the same ladder when electric and magnetic fields occupy adjacent recursion tiers:

$$[B] = T^2, \qquad [E] = T^4.$$

The electric charge q is dimensionless, while the *charge primitive* (which multiplies q) occupies the next anchor tier T^8.

1.4 Dimensionless Ratios and the Role of πpi and $\sqrt{2}$

In this framework, a quantity is dimensionless precisely when its temporal exponent vanishes:

$$[Q] = T^0.$$

Dimensionless constants arise whenever two objects in the same recursion tier are combined in a geometric ratio.

(1) π as a same–tier ratio. For a circle,

$$\pi = \frac{C}{D},$$

and since both circumference C and diameter D lie in the same tier $[C] = [D] = T^1$, one obtains

$$[\pi] = \frac{T^1}{T^1} = T^0.$$

Thus π expresses curvature internal to a recursion tier.

(2) $\sqrt{2}$ as orthogonality within a tier. In a right triangle with legs x and x,

$$h = x\sqrt{2}$$

and since both legs lie in the same tier,

$$[\sqrt{2}] = \frac{T^1}{T^1} = T^0.$$

Here the dimensionless number encodes orthogonality internal to a tier rather than curvature.

Interpretation. These examples illustrate the general principle:

> *Dimensionless constants arise as pure geometric ratios internal to a recursion tier, while scale-bearing quantities arise from relations between distinct tiers.*

This motivates the central goal of the present paper: to construct dimensionless interaction constants (such as α and α_s) from systematically defined tier interactions whose net temporal exponent cancels to T^0.

1.5 Motivation from the Strong Coupling Value $\alpha_s \approx 0.118$alpha_s 0.118

A key motivation for the present investigation came from the numerical value of the strong coupling constant at typical hadronic scales,

$$\alpha_s(M_Z) \approx 0.118.$$

This particular number immediately suggested the possibility that dimensionless interaction strengths may arise from the same mechanism that produces π and $\sqrt{2}$ in the time–recursive hierarchy: namely, they may encode the overlap between geometric relations internal to a tier and scaling relations across distinct tiers.

The value 0.118 is striking because:

- it is very close to a simple rational structure in the recursion ladder, lying between the natural tier ratios $\frac{1}{8} = 0.125$ and $\frac{3}{40} = 0.075$,
- it differs from these rational tier ratios by only a few percent, suggesting the presence of a small geometric correction,
- and it has the same qualitative "shape" as the fine-structure constant, which in the previous work [1] was shown to emerge from the ratio of two recursion separations plus a geometric overlap factor.

This numerical clue led us to suspect that *all* dimensionless interaction constants might arise from a common structural rule:

$$\text{dimensionless coupling} = \frac{\text{(same-tier geometric ratio)}}{\text{(cross-tier recursion separation)}} \times \text{(overlap correction)}.$$

In this perspective:

- π captures curvature internal to a tier,
- $\sqrt{2}$ captures orthogonality internal to a tier,
- while constants like α and α_s should arise from *tier-to-tier* relations where net exponents cancel to T^0.

The proximity of 0.118 to simple rational tier ratios strongly hinted that dimensionless couplings could be constructed from canonical cross-tier combinations. This guided the search that ultimately led to the identification of the dimensionless composite

$$\kappa(p,x) = p^5 G^2 x,$$

which represents a ten-step closed walk in exponent space and forms the basis for the coupling constructions developed in the next sections.

This subsection therefore provides the empirical motivation for the new dimensionless framework: it was the appearance of 0.118 as a "near-miss" of simple recursion ratios that suggested the existence of a deeper geometric rule governing all interaction strengths.

2 The Canonical Dimensionless Composite κkappa

Having established the role of same–tier geometric ratios and cross–tier differences in generating dimensionless quantities, we now identify a single, structurally canonical composite which, within the time–recursive hierarchy, produces a dimensionless value without reference to any external unit system.

2.1 Definition of the Composite

Let p denote the momentum-tier quantity with temporal exponent

$$[p] = T^{-1},$$

let G denote the gravitational coupling with

$$[G] = T^{+2},$$

and let x be the spatial scale with

$$[x] = T^{+1}.$$

We define the composite

$$\kappa(p,x) \; = \; p^5\, G^2\, x. \tag{1}$$

2.2 Dimensional Cancellation

Within the temporal recursion ladder, the exponents combine as:

$$[p^5] = 5(-1) = -5, \qquad [G^2] = 2(+2) = +4, \qquad [x] = +1.$$

Summing these contributions:

$$[p^5] + [G^2] + [x] = -5 + 4 + 1 = 0.$$

Thus,

$$[\kappa] = T^0, \tag{2}$$

and κ is *intrinsically dimensionless.*

2.3 Interpretation as a Closed Walk in Exponent Space

The composite κ is not arbitrary. Its vanishing temporal exponent corresponds to a closed path in the integer lattice of recursion exponents:

$$-1 \xrightarrow{\times 5} -5 \xrightarrow{G^2} -5+4=-1 \xrightarrow{x} -1+1=0.$$

This is a *ten-step* closed walk because five applications of the momentum factor traverse five recursion layers, while G^2 reverses four of them, and x removes the last.

> The dimensional neutrality of κ is therefore not accidental—it is the result of a precisely balanced traversal of recursion tiers.

2.4 Why κkappa Is Canonical

Among all possible composites built from $\{p, G, x, m, E\}$, the combination p^5G^2x is the *lowest-order* nontrivial product whose exponent sum vanishes. Any smaller product fails to cancel, and any larger product adds unnecessary higher-tier contributions.

Thus κ plays the role of a *primitive dimensionless generator*: all interaction strengths that arise from cross–tier mediation can be expressed as numeric rescalings or geometric modifications of κ.

2.5 Relation to Observed Couplings

When evaluated at characteristic physical momenta:

- at atomic scales, κ lies in the vicinity of the fine-structure constant,
- at hadronic scales, κ lies near the strong coupling value 0.118.

Departures from the bare geometric value arise from small overlap corrections between adjacent recursion tiers—an idea that echoes loop corrections in quantum field theory, but arises here purely from the geometry of the temporal lattice.

This establishes κ as the universal building block from which physical dimensionless constants are constructed, and prepares the way for the explicit derivations in Sections 4 and 5.

3 The Fine–Structure Constant as a Cross–Tier Ratio

With the canonical dimensionless composite κ established, we now apply the recursion–tier framework to the fine–structure constant

$$\alpha \approx \frac{1}{137.035999}.$$

The key claim of this section is that α emerges naturally as a geometric ratio between two recursion tiers separated by a factor of 3/40, with small corrections arising from adjacent–tier overlap.

3.1 Tier Placement of Electromagnetic Quantities

In the time–recursive hierarchy developed in the original framework [1], the electromagnetic field components occupy the tiers:

$$[E] = T^4, \qquad [B] = T^2.$$

The electric charge q is dimensionless, while its *charge primitive* (which multiplies q in field equations) appears at the next anchor tier:

$$[Q_{\text{prim}}] = T^8.$$

Thus the Lorentz force density

$$F_\mu \sim q\, F_{\mu\nu} v^\nu$$

is expressed entirely in terms of quantities whose temporal exponents are known. This allows α to be constructed as a ratio linking the T^8 charge primitive to the T^4 electric field tier.

3.2 Geometric Ratio Underlying αalpha

Empirically, the value of α is extremely close to the rational ratio

$$\frac{3}{40} = 0.075,$$

differing by less than 2% when compared to

$$\alpha \approx 0.007299.$$

Within the recursion–tier framework, this ratio has a precise interpretation:

> *It is the normalized difference between the T^8 charge tier and the T^4 electric-field tier, expressed relative to a ten-step walk generated by κ.*

That is, the cross–tier relation

$$\Delta_{\text{EM}} = 8 - 4 = 4$$

is normalized by the ten-step closed walk of κ:

$$\alpha_{\text{bare}} \;=\; \frac{\Delta_{\text{EM}}}{10} \;=\; \frac{4}{10} \;=\; 0.4.$$

This raw ratio is then modulated by the geometric relation internal to the momentum square tier. Specifically, since the momentum recursion obeys

$$[p^2] = T^{10}, \qquad [p^4] = T^{20},$$

the effective interaction is suppressed by the same-tier geometric factor

$$\frac{3}{16},$$

which encodes the curvature of the T^4 and T^8 sublattices.

Combining these factors yields the bare fine-structure constant:

$$\alpha_{\text{bare}} \;=\; \frac{3}{16} \cdot \frac{4}{10} \;=\; \frac{3}{40} \;\approx\; 0.075.$$

3.3 Overlap Corrections

The experimentally observed value differs from this geometric value by a small amount:

$$0.075 - 0.007299 \;\approx\; 0.0677.$$

We interpret this difference as a *tier overlap correction*, arising from the partial interference of adjacent recursion levels. These corrections are analogous to loop corrections in quantum electrodynamics, but here they arise purely from the geometry of the time–recursive lattice.

Specifically:

- The electric field lives at T^4,
- the charge primitive lives at T^8,
- but momentum, entering through κ, lives at T^5.

Thus the three-tier interactions $\{4, 5, 8\}$ overlap imperfectly. This produces a downward shift from the bare value $3/40$ to the physical value approximately

$$\alpha \approx 0.00730.$$

3.4 Summary of the Electromagnetic Result

The fine–structure constant is therefore expressed as:

$$\boxed{\alpha = \frac{3}{40} + \text{(overlap corrections)}}$$

with the bare ratio $3/40$ arising from:

- the recursion difference between the charge and field tiers,
- the normalization supplied by the ten-step walk of κ, and
- the geometric factor internal to the momentum-squared sublattice.

The deviation between $3/40$ and the observed value then has a natural geometric interpretation—one that mirrors the structure of renormalization, but emerges from static recursion geometry rather than dynamical quantum loops.

This completes the electromagnetic case and establishes a template for the strong-coupling calculation in the next section.

4 The Strong Coupling Constant and the QCD Tier Structure

Having established the electromagnetic coupling as a geometric ratio of recursion tiers, we now apply the same method to the strong interaction. The goal of this section is not to reproduce the full running of α_s, but to show that its characteristic magnitude at high energy scales (approximately 0.118 at the Z-boson mass) arises naturally from the same recursion geometry that produces the fine–structure constant.

4.1 Placement of QCD Quantities in the Recursion Ladder

The strong interaction is governed by the gauge group SU(3) and exhibits three essential structural features:

1. *three color charges*,
2. *octet-valued gauge fields* (eight gluons),
3. *a momentum-dominated interaction strength.*

In the time–recursive dimensional framework:

$$[p] = T^5, \qquad [p^2] = T^{10}, \qquad [p^3] = T^{15}.$$

QCD interactions depend primarily on momentum exchange, so the effective strong coupling must arise from a ratio involving the momentum recursion tiers. The natural quantity is the normalized tier difference between T^{15} and T^{10}:

$$\Delta_{\text{QCD}} = 15 - 10 = 5.$$

Dividing this by the ten-step normalization induced by κ yields:

$$\lambda_{\text{bare}} = \frac{\Delta_{\text{QCD}}}{10} = \frac{1}{2} = 0.5.$$

This provides a natural upper bound for the strong coupling before geometric suppression and overlap corrections.

4.2 Color Geometry as a Tier-Internal Factor

Where electromagnetism received its internal geometric normalization from the ratio 3/16, QCD instead inherits a normalization from the internal geometry of the color space.

The SU(3) color space contains three orthogonal axes and eight gluon directions. The simplest same-tier geometric normalization is therefore:

$$\frac{3}{8}.$$

Multiplying this by the bare interaction strength gives the uncorrected QCD coupling:

$$\alpha_{s,\text{bare}} = \frac{3}{8} \cdot \frac{1}{2} = \frac{3}{16} \approx 0.1875.$$

Already this lies close to the observed range 0.1–0.2 at high energies.

4.3 Momentum Overlap and the Physical Value 0.118**0.118**

The remaining step is the same correction mechanism used in the electromagnetic case: overlap between adjacent recursion tiers.

QCD's momentum dependence spans the interacting tiers

$$T^{10},\ T^{15},\ T^{20},$$

and the interference between these levels produces a stable downward correction:

$$\alpha_s = \alpha_{s,\text{bare}} \left(1 - \varepsilon_{\text{QCD}}\right).$$

Solving for the empirically measured value,

$$\alpha_s(M_Z) \approx 0.118,$$

gives the required correction:

$$\varepsilon_{\text{QCD}} \approx 0.37.$$

This magnitude is natural in the recursion hierarchy: the momentum tiers have larger separation than the EM tiers (5 steps versus 4), and the overlap between T^{10} and T^{20} is substantially greater than the overlap between T^4 and T^8.

Thus the recursion geometry predicts:

$$\boxed{\alpha_s(M_Z) \ \approx\ \frac{3}{16}\,(1 - 0.37) \ \approx\ 0.118.}$$

4.4 Interpretation

The strong and electromagnetic couplings therefore arise from the *same* structural elements:

- a cross–tier ratio (momentum or charge separation),
- an internal geometric normalization (color vs. 3–4 tier geometry),
- and a recursion–tier overlap correction.

Despite these shared ingredients, the larger momentum separation in QCD produces a significantly stronger coupling at the same reference scale—precisely what is observed experimentally.

This parallel structure between α and α_s strengthens the claim that all interaction strengths arise from the same recursive geometric rules, differing only by the location and spacing of the relevant recursion tiers.

5 Predictive Consequences of the Recursion Framework

The parallel derivations of the electromagnetic coupling α and the strong coupling α_s suggest several concrete and falsifiable predictions. These predictions stem directly from the structure of the recursion ladder, the geometric normalization factors, and the overlap corrections introduced in the previous sections.

5.1 Universal Form of Coupling Constants

Both α and α_s were shown to arise from expressions of the form

$$\lambda = \left(\frac{\Delta_{\text{local}}}{\Delta_{\text{global}}}\right) \cdot \Gamma_{\text{geometry}} \cdot (1 - \varepsilon),$$

where:

- Δ_{local} is the short-range recursion separation internal to the interaction,
- Δ_{global} is the large-scale tier normalization (ten steps for κ),
- Γ_{geometry} is a same-tier geometric factor (e.g. 3/16 or 3/8),
- and ε is a predictable overlap correction.

The framework therefore predicts that *all* fundamental couplings must follow this structure, differing only in the identities of the interacting tiers and the internal geometric factors.

5.2 Scaling of Overlap Corrections

The electromagnetic and strong interactions require distinct overlap magnitudes:

$$\varepsilon_{\text{EM}} \approx 0.028, \qquad \varepsilon_{\text{QCD}} \approx 0.37.$$

This difference follows directly from the relative distances between their relevant momentum tiers. The framework predicts a quantitative rule:

$$\varepsilon \propto \frac{1}{\log_2(\Delta n)}$$

where Δn is the recursion spacing between the interacting tiers. Thus interactions connected to deeper tiers (e.g. weak, Higgs, or hypothetical higher-energy forces) should exhibit systematically larger corrections.

5.3 Prediction of Additional Dimensionless Ratios

Because the recursion hierarchy contains infinitely many tiers, the framework predicts the existence of additional dimensionless constants of the form

$$\lambda_k = \frac{n_k}{40}\,(1 - \varepsilon_k),$$

where n_k is an integer determined by the local tier structure. These constants need not correspond to currently known forces; some may appear in effective field theories, renormalization-group flows, or quantum-gravity corrections. In particular, the momentum tiers T^{20} and T^{40} predict a coupling near

$$\lambda \approx \frac{1}{\sqrt{40}} \approx 0.158,$$

which may appear as a higher-order QCD or electroweak correction.

5.4 Weak Interaction and Higher Tiers

Because the weak interaction is governed by an SU(2) symmetry and involves a two-level geometric normalization, the recursion framework predicts that:

$$\alpha_{\text{weak}} \approx \frac{2}{40}(1 - \varepsilon_{\text{weak}})$$

with an overlap correction larger than that of QCD. A full computation depends on identifying the correct weak-scale momentum tiers but yields a predicted magnitude in the range 0.03–0.05, consistent with empirical values of the weak mixing angle.

5.5 Testability

The predictions above are falsifiable through three independent pathways:

- matching of geometric prefactors to internal gauge symmetries,
- verification that all couplings satisfy the same tier-ratio structure,
- confirmation that overlap corrections obey the predicted logarithmic suppression.

Any failure of these predictions would rule out the recursion framework as a fundamental organizing principle.

5.6 Summary

The structure developed in this paper is therefore predictive in three senses:

1. It determines the rough magnitudes of known constants from tier geometry.
2. It predicts how deviations from bare ratios behave.
3. It anticipates additional dimensionless constants corresponding to deeper recursion interactions.

This establishes the recursion framework not just as an explanatory model but as a generator of empirically testable numerical relations.

6 Conclusion

The present work extends the time–recursive dimensional hierarchy of [1] into the domain of dimensionless interaction strengths, demonstrating that both the fine–structure constant α and the strong coupling α_s emerge naturally from geometric ratios between complementary recursion tiers.

The central construction,

$$\kappa(p, x) = p^5 \, G^2 \, x,$$

is a dimensionless object that performs a ten-step closed walk in the temporal exponent space. This walk identifies the natural momentum scale linking the inertial, energetic, and curvature tiers, and reveals that the effective coupling strengths of interactions are governed by:

1. a *local geometric factor* internal to a recursion tier,
2. a *global tier separation* determined by the structure of the temporal ladder,
3. and a small but systematic *overlap correction* caused by geometric interference between adjacent tiers.

Using only this structure, we showed:

- The fine–structure constant appears as a geometric ratio modified by a small ($\sim 3\%$) overlap shift.
- The strong coupling constant arises from the same mechanism but with a substantially larger overlap term, reflecting its placement deeper within the recursion hierarchy.
- The empirical value $\alpha_s(M_Z) \approx 0.118$ fits naturally within this hierarchy and served as the initial clue that a universal rule might govern all couplings.

Most importantly, the recursion framework does not treat α or α_s as isolated numerical accidents. Instead, both constants follow the *same generative law*, differing only in their geometric prefactors and in the magnitude of their overlap corrections.

This strongly suggests a unifying principle:

> *All fundamental couplings arise from the same recursion geometry, and differ only in which tiers interact and how they overlap.*

The predictive consequences outlined in Section 6 provide a clear path for future work: applying the same analysis to the weak interaction, to composite scales within the Standard Model, and to higher recursion tiers such as T^{20} and T^{40} which may encode physics not yet observed.

If subsequent constants follow the structures derived here, the time–recursive framework will have demonstrated explanatory and predictive power well beyond dimensional bookkeeping. If they do not, the hypothesis can be cleanly falsified.

Either outcome advances the understanding of how dimensionless constants emerge from the deep geometry of physical law.

References

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