

A Recursive Time-Dimensional Framework for Physical Interactions

Version 2.0

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Abstract

This work develops a dimensional framework in which all physical quantities are expressed as powers of a single primitive unit of time. The structure is based on a recursive hierarchy of “anchor” exponents

$$T^1, T^2, T^4, T^8, \dots,$$

generated by the map $n \mapsto 2n$. Conventional kinematic quantities such as spatial length, velocity, and acceleration are recovered by placing them at intermediate positions between these anchor tiers. In particular, consistency with the standard relations $v = L/t$ and $a = L/t^2$ requires the assignments

$$[t] = T^1, \quad [L] = T^3, \quad [v] = T^2, \quad [a] = T^1.$$

This refinement restores the correct dimensional relations of classical kinematics while preserving the recursive structure of the anchor hierarchy.

Building from this base, gravitational and electromagnetic laws are examined in terms of the same time-only exponents. The weak-field relation $g \sim GM/R^2$ enforces the constraint $[G][M] = T^7$, and together with the kinematic assignments yields natural placements for mass, spatial geometry, and gravitational coupling within the recursive structure. The resulting hierarchy provides a unified dimensional organization in which space, mass, acceleration, forces, and interaction constants arise as distinct temporal exponents generated by a single primitive.

1 Introduction

The dimensional structure of physics traditionally relies on several independent base units: length, mass, time, charge, temperature, and others. While indispensable for measurement, these units obscure possible underlying relationships between physical quantities. The coexistence of incompatible dimensional foundations in kinematics, gravity, electrodynamics, and quantum field theory hinders attempts at unification.

In this work we explore a framework in which *time* is taken as the sole primitive unit, and all other quantities—spatial, inertial, dynamical, or field-theoretic—are constructed as powers of this primitive. A recursive map on exponents produces a global hierarchy of “anchor tiers,”

$$T^1, T^2, T^4, T^8, \dots,$$

which provide the backbone for a dimensional classification of physical interactions. Intermediate exponents naturally arise between these tiers, allowing spatial and inertial quantities to be incorporated without introducing new base units.

A central motivation for this approach is the possibility that the apparent diversity of dimensional types in physics reflects a deeper, time-generated structure. By reducing the dimensional basis to a single primitive, we expose relationships that are otherwise hidden by the conventional SI decomposition. The goal is not to redefine measurement practice, but to uncover an underlying generative mechanism that organizes fundamental interactions.

2 Foundations of the Time-Primitive Framework

The framework begins with a single primitive unit of time, denoted T . All other quantities are assigned dimensions of the form T^n for some real exponent n . This section introduces the temporal primitive, the recursive anchor hierarchy, and the way classical kinematic quantities are recovered within this structure.

2.1 The Temporal Primitive

We denote the fundamental primitive by T . Any physical quantity Q in the theory has a dimensional assignment

$$[Q] = T^n.$$

The exponent n measures the degree to which Q derives from the primitive time unit. In contrast to conventional physics, no base units for length, mass, or charge are assumed at the foundational level. These quantities, if they appear, must be constructed as derived exponents of T .

2.2 The Recursive Anchor Hierarchy

A key structural element of the framework is a recursive doubling map on exponents:

$$n \mapsto 2n.$$

Applying this map iteratively produces a global sequence of *anchor tiers*,

$$T^1, T^2, T^4, T^8, T^{16}, \dots$$

These tiers form a distinguished lattice within the space of temporal exponents. They represent preferred structural positions into which interaction strengths and dynamical roles may naturally fall.

The anchor tiers do *not* exhaust the possible exponents. Physical quantities can occupy intermediate values (e.g. T^3), but the recursive map generates an organizational backbone that plays a central role in the structure of the theory.

2.3 Recovering Classical Kinematic Dimensions

Unlike earlier formulations, we no longer assert that velocity or acceleration occupy specific anchor tiers by fiat. Instead, we require that classical kinematic relations hold identically when rewritten in the time-exponent language.

Let the dimension of physical time be

$$[t] = T^1.$$

Let spatial length L be assigned a general exponent T^λ . Kinematic relations then impose strict consistency conditions:

$$[v] = \frac{[L]}{[t]} = T^{\lambda-1}, \quad [a] = \frac{[L]}{[t]^2} = T^{\lambda-2}.$$

To maintain the empirical fact that velocity is dimensionally “between” time and its first recursive anchor T^2 , we require

$$[v] = T^2.$$

This determines λ :

$$\lambda - 1 = 2 \quad \Rightarrow \quad \lambda = 3.$$

Thus we obtain the unique assignments

$$[L] = T^3, \quad [v] = T^2, \quad [a] = T^1.$$

This simultaneously preserves all classical kinematic identities and resituates length as a derived quantity lying between the T^2 and T^4 anchor tiers. Notably, time and acceleration share the same exponent T^1 , reflecting the fact that acceleration is a second temporal derivative of position.

These kinematic assignments form the base upon which gravitational, relativistic, and electromagnetic relations will be evaluated in subsequent sections.

3 Recursive Hierarchy of Dynamical Roles

The kinematic assignments of Section 2 fix how time, length, velocity, and acceleration are represented in the time-exponent language. We now develop the more general recursive structure that organizes all dimensional roles, independent of any specific physical interpretation.

3.1 Global Ladder of Anchor Exponents

The recursive map

$$n \mapsto 2n$$

acting on the temporal exponent generates a global ladder of anchor tiers,

$$T^1, T^2, T^4, T^8, T^{16}, \dots \tag{1}$$

For clarity, we define

$$Q_0 = T^1, \quad Q_k = T^{2^k} \text{ for } k \geq 0, \tag{2}$$

so that the anchor sequence can be written compactly as

$$\{Q_k\}_{k=0}^{\infty} = \{T^{2^k}\}.$$

This ladder plays a purely structural role. It specifies a discrete set of preferred exponents that can host distinct dynamical or interaction “tiers”. Intermediate exponents such as T^3 or T^5 are not forbidden; rather, they appear as derived quantities lying between anchors. The kinematic assignments

$$[t] = T^1, \quad [L] = T^3, \quad [v] = T^2, \quad [a] = T^1$$

provide an explicit example of how such intermediate exponents coexist with the anchor hierarchy.

3.2 Formal Properties of the Recursive Structure

Viewed purely as a sequence of exponents, the anchor ladder exhibits several simple but important properties.

First, the exponents are strictly increasing:

$$1 < 2 < 4 < 8 < 16 < \dots,$$

so that no two anchor tiers share the same dimensional role. Each Q_k represents a distinct class of temporal scaling.

Second, the sequence is multiplicatively self-similar. For any integers k and m ,

$$Q_{k+m} = T^{2^{k+m}} = (T^{2^k})^{2^m} = (Q_k)^{2^m}. \quad (3)$$

Higher tiers can therefore be expressed as scaled powers of lower ones. This self-similarity is the algebraic origin of the recursive patterns that later appear in the placement of interaction strengths and field quantities.

Third, the recursion preserves ordering. If $a < b$ at the level of exponents, then

$$2a < 2b,$$

so repeated application of the doubling map cannot collapse or invert the hierarchy. The ladder is stable under its own generative rule.

Finally, the set of anchor exponents is infinite but countable, providing an unbounded supply of distinct tiers onto which different physical roles can be mapped. The later sections of this paper use only the lowest few tiers (T^1 through T^8), but nothing in the construction prevents higher-order interactions from occupying T^{16} or beyond.

3.3 Local Four-Stage Cycle at Each Tier

The global ladder $\{Q_k\}$ organizes exponents, but physical theories typically distinguish between several roles *within* a given dimensional level: raw units, fields, forces, and extreme or boundary configurations. To capture this, we associate with each anchor tier Q_k a local four-stage refinement,

$$\mathcal{T}_k = \{Q_k^{(0)}, Q_k^{(1)}, Q_k^{(2)}, Q_k^{(3)}\}, \quad (4)$$

interpreted abstractly as

$$\begin{aligned} Q_k^{(0)} &: \text{Unit,} \\ Q_k^{(1)} &: \text{Field,} \\ Q_k^{(2)} &: \text{Force,} \\ Q_k^{(3)} &: \text{Anomaly or boundary configuration.} \end{aligned} \quad (5)$$

At this stage, these labels are purely structural:

- The *Unit* stage represents the bare dimensional object at that tier, stripped of any geometric or dynamical structure.
- The *Field* stage corresponds to an extended or geometric configuration of that unit—for example, a distributed quantity over space or spacetime.
- The *Force* stage captures the effective interaction generated when the field acts on suitable sources or test objects.

- The *Anomaly* stage denotes an extreme, singular, or limiting configuration at the edge of the regime defined by Q_k , such as critical points or breakdown scales.

The transition

$$Q_k^{(0)} \rightarrow Q_k^{(1)} \rightarrow Q_k^{(2)} \rightarrow Q_k^{(3)}$$

is intended to be universal: every tier in the anchor ladder admits the same conceptual refinement pattern. This produces a “tree of trees”: a global sequence of exponents T^{2^k} , and at each exponent a local four-stage cycle of unit, field, force, and anomaly.

Intermediate exponents such as T^3 inherit this structure indirectly. They are positioned relative to the nearest anchors by their algebraic relations (for example, $[L] = T^3$ lies between T^2 and T^4), and their roles are determined by how they participate in kinematic and dynamical equations that involve the anchor tiers.

In the sections that follow, specific physical quantities—including mass, gravitational acceleration, the gravitational constant, and electromagnetic fields—are placed onto this scaffold. The test of the framework is whether independent physical laws select consistent positions within the same recursive time-dimensional hierarchy.

4 Gravitational Laws in the Time-Exponent Framework

With the kinematic structure and recursive hierarchy in place, we now examine how gravitational laws fit into the same time-exponent language. The goal is to determine where mass, gravitational acceleration, the gravitational constant, and force sit in the recursive structure, using only standard relations rewritten in terms of powers of T .

4.1 Dimensional Inputs from Kinematics

From Section 2.3 we already have

$$[t] = T^1, \quad [L] = T^3, \quad [v] = T^2, \quad [a] = T^1.$$

Gravitational acceleration g is an acceleration and therefore shares the same dimension as a :

$$[g] = T^1.$$

Radial distances r and R appearing in gravitational laws are spatial lengths, so

$$[r] = [R] = [L] = T^3, \quad [r^2] = [R^2] = T^6.$$

The remaining gravitational quantities—mass M , the gravitational constant G , and force F —are initially left with unknown exponents:

$$[M] = T^\mu, \quad [G] = T^\gamma, \quad [F] = T^\phi.$$

These exponents will be fixed by demanding consistency of the usual gravitational relations in the time-exponent framework.

4.2 Weak-Field Relation and the Product $[G][M][G][M]$

In the weak-field limit of general relativity, the gravitational acceleration outside a spherically symmetric mass distribution satisfies

$$g \sim \frac{GM}{R^2}. \tag{6}$$

Rewriting this in terms of time exponents, we substitute

$$[g] = T^1, \quad [G] = T^\gamma, \quad [M] = T^\mu, \quad [R^2] = T^6,$$

to obtain

$$T^1 \sim \frac{T^\gamma T^\mu}{T^6} = T^{\gamma+\mu-6}. \quad (7)$$

Equality of exponents then imposes the constraint

$$\gamma + \mu - 6 = 1 \implies \gamma + \mu = 7. \quad (8)$$

Thus the weak-field relation does not fix $[G]$ and $[M]$ individually, but it forces their product to occupy the tier

$$[G][M] = T^7. \quad (9)$$

Any consistent placement of G and M in the recursive hierarchy must respect this combined exponent.

4.3 Newtonian Gravity and Inertial Force

The Newtonian gravitational force between two equal masses $M = m$ at separation r is

$$F = \frac{GM^2}{r^2}. \quad (10)$$

In the time-exponent framework, this becomes

$$[F] = \frac{[G][M]^2}{[r]^2} = \frac{T^\gamma T^{2\mu}}{T^6} = T^{\gamma+2\mu-6}. \quad (11)$$

Independently, inertial dynamics requires

$$F = Ma, \quad (12)$$

which implies

$$[F] = [M][a] = T^\mu T^1 = T^{\mu+1}. \quad (13)$$

Equating the exponents obtained from Eqs. (11) and (13) yields

$$\mu + 1 = \gamma + 2\mu - 6 \implies \gamma = 7 - \mu. \quad (14)$$

This is precisely the relation already obtained from the weak-field constraint (8), confirming that the Newtonian force law and the relativistic weak-field expression are mutually consistent in the time-exponent framework.

At this stage, the exponents μ and γ remain underdetermined: the two gravitational laws agree with each other but do not yet single out a unique tier for mass or G . The remaining freedom is associated with the overall choice of which anchor tier should host the force dimension.

4.4 Fixing the Force Tier on the Anchor Ladder

The recursive structure introduced in Section 3 singles out T^4 as the lowest anchor tier above velocity (T^2) and below higher-order interaction tiers such as T^8 . It is natural to associate this first “higher” anchor with the role of force in the hierarchy:

$$[F] = T^4. \quad (15)$$

Combining Eq. (15) with the inertial relation (13),

$$[F] = T^{\mu+1},$$

gives

$$\mu + 1 = 4 \implies \mu = 3. \quad (16)$$

Thus mass occupies the same exponent as spatial length:

$$[M] = T^3. \quad (17)$$

Substituting $\mu = 3$ into Eq. (8) or Eq. (14) then yields

$$\gamma + 3 = 7 \implies \gamma = 4, \quad (18)$$

so that

$$[G] = T^4. \quad (19)$$

With these assignments,

$$[M] = T^3, \quad [G] = T^4, \quad [F] = T^4, \quad [g] = T^1,$$

both gravitational laws are automatically satisfied:

- Weak-field relation:

$$[g] = \frac{[G][M]}{[R]^2} = \frac{T^4 T^3}{T^6} = T^1.$$

- Newtonian force law with $M = m$:

$$[F] = \frac{[G][M]^2}{[r]^2} = \frac{T^4 T^6}{T^6} = T^4.$$

Inertial dynamics is likewise consistent:

$$[F] = [M][a] = T^3 T^1 = T^4.$$

4.5 Resulting Gravitational Hierarchy

The combined analysis leads to a simple and coherent placement of gravitational quantities within the recursive time-dimensional structure:

- Time and acceleration share the base exponent:

$$[t] = T^1, \quad [a] = T^1, \quad [g] = T^1.$$

- Velocity occupies the intermediate tier between T^1 and T^4 :

$$[v] = T^2.$$

- Spatial length and mass share the same derived geometric tier:

$$[L] = T^3, \quad [r] = [R] = T^3, \quad [M] = T^3.$$

- Force and the gravitational constant occupy the first higher anchor tier:

$$[F] = T^4, \quad [G] = T^4.$$

In this picture, mass is not an independent primitive but a geometric quantity sharing the same exponent as spatial length. Gravitational coupling G lives on the same anchor tier as force, reflecting its role as a universal measure of how strongly geometric mass distributions curve the temporal hierarchy into acceleration.

The next sections extend this analysis to relativistic energy and electromagnetic interactions, testing whether those laws also select consistent positions on the same recursive ladder of time exponents.

5 Relativistic Energy in the Time-Exponent Framework

With the gravitational sector fixed, we now examine how relativistic energy fits into the same time-exponent structure. The key question is whether the relativistic mass–energy relation

$$E = Mc^2 \quad (20)$$

selects a unique exponent for energy that is consistent with both classical mechanics and the gravitational relations already analyzed.

5.1 Energy from the Mass–Energy Relation

In the time-exponent framework, we have already established that

$$[M] = T^3, \quad [c] = [v] = T^2.$$

Substituting these assignments into the mass–energy relation (20) gives

$$[E] = [M][c]^2 = T^3 \cdot (T^2)^2 = T^3 \cdot T^4 = T^7. \quad (21)$$

Thus energy occupies the exponent tier

$$[E] = T^7. \quad (22)$$

This result is notable for two reasons. First, it places energy at a derived tier between the anchor levels T^4 and T^8 , analogous to the way length and mass occupy the derived tier T^3 between the anchors T^2 and T^4 . Second, the exponent T^7 coincides with the product $[G][M]$ obtained earlier from the weak-field gravitational relation, suggesting a deep link between energy, mass, and gravitational coupling in the time-exponent hierarchy.

5.2 Consistency with Work and Gravitational Potential Energy

Classically, energy also appears as mechanical work,

$$E = FL, \quad (23)$$

where F is force and L is a characteristic length or displacement. In the time-exponent framework, we have

$$[F] = T^4, \quad [L] = T^3,$$

so

$$[E] = [F][L] = T^4 \cdot T^3 = T^7. \quad (24)$$

Thus the work relation reproduces the same exponent for energy as the mass–energy relation (20).

Gravitational potential energy provides a third, independent check. The Newtonian potential energy of two masses M and m separated by distance r is

$$E_{\text{grav}} \sim -\frac{GMm}{r}. \quad (25)$$

Assigning

$$[G] = T^4, \quad [M] = [m] = T^3, \quad [r] = T^3,$$

we obtain

$$[E_{\text{grav}}] = \frac{[G][M][m]}{[r]} = \frac{T^4 \cdot T^3 \cdot T^3}{T^3} = T^{4+3+3-3} = T^7. \quad (26)$$

The agreement of these three independent constructions,

- relativistic mass–energy equivalence,
- mechanical work $E = FL$, and
- gravitational potential energy $E_{\text{grav}} \sim GMm/r$,

confirms that the energy tier is uniquely and consistently fixed at

$$[E] = T^7 \quad (27)$$

within the recursive time-dimensional framework.

5.3 Relation to the Gravitational Product $[G][M][G][M]$

From the weak-field analysis in Section 4.2, we found that gravitational acceleration satisfies

$$[g] \sim \frac{[G][M]}{[R]^2},$$

leading to the constraint

$$[G][M] = T^7.$$

With the assignments

$$[G] = T^4, \quad [M] = T^3,$$

this becomes

$$[G][M] = T^4 \cdot T^3 = T^7.$$

We now see that

$$[E] = [G][M]. \quad (28)$$

In other words, the dimensional placement of energy coincides exactly with the combined gravitational quantity GM . In conventional general relativity, the combination GM controls both the gravitational potential and the curvature scale associated with a mass distribution. In the present framework, this correspondence is elevated to a dimensional identity: the energy tier T^7 is the same tier that hosts the basic gravitational coupling of mass to geometry.

This reinforces the interpretation of T^7 as an “energy–curvature” tier in the time-exponent hierarchy: it is simultaneously the home of relativistic energy and the effective gravitational strength generated by a mass.

5.4 Planck Energy and the Quantum Anchor Tier

The time-exponent placement of energy also constrains the dimensional role of quantum constants. Consider the Planck energy,

$$E_P = \sqrt{\frac{\hbar c^5}{G}}, \quad (29)$$

which in conventional units is constructed from the reduced Planck constant \hbar , the speed of light c , and the gravitational constant G . Let the time-exponent of \hbar be

$$[\hbar] = T^\eta.$$

Substituting the known assignments

$$[E_P] = [E] = T^7, \quad [c] = T^2, \quad [G] = T^4,$$

we have

$$[E_P]^2 = [\hbar] [c]^5 [G]^{-1} = T^\eta \cdot T^{10} \cdot T^{-4} = T^{\eta+6}. \quad (30)$$

Since $[E_P]^2 = [E]^2 = T^{14}$, exponent equality requires

$$14 = \eta + 6 \implies \eta = 8. \quad (31)$$

Therefore,

$$[\hbar] = T^8. \quad (32)$$

This result is structurally significant. The exponent T^8 is precisely the next anchor tier in the global ladder,

$$T^1, T^2, T^4, T^8, T^{16}, \dots$$

Energy resides at the derived level T^7 , just below this anchor, in the same way that length and mass reside at the derived level T^3 just below the force anchor T^4 . The reduced Planck constant \hbar thus occupies a special position as a *quantum anchor*: it lives exactly on the next doubling tier above the gravitational force level and one step above the energy tier.

In this view, quantum mechanics is not introduced as an independent dimensional sector, but as a higher anchor in the same time-recursive hierarchy. The placement

$$[E] = T^7, \quad [\hbar] = T^8$$

captures the familiar relationship between energy scales and quantum action: energy sits immediately below the quantum anchor, and Planck-scale combinations such as E_P emerge from the interplay of \hbar , c , and G within this shared time-based ladder.

5.5 Momentum and Intermediate Tiers

For completeness, we briefly note the dimensional placement of relativistic momentum. From the standard relation

$$p = \frac{E}{c}, \quad (33)$$

we obtain

$$[p] = \frac{[E]}{[c]} = \frac{T^7}{T^2} = T^5. \quad (34)$$

Thus momentum occupies the intermediate exponent T^5 , lying between the force tier T^4 and the energy tier T^7 . This mirrors the role of velocity at T^2 , which interpolates between the base tier T^1 (time/acceleration) and the force tier T^4 .

The appearance of momentum at T^5 suggests that the “gap” exponents between anchors and derived tiers (such as T^5 between T^4 and T^7) are naturally populated by dynamical quantities that mediate between force and energy. A full exploration of these intermediate tiers, including stress, pressure, and energy density, is left to future work, but the placement of momentum already indicates that the recursive ladder can accommodate them without additional primitives.

5.6 Summary of the Energy and Quantum Structure

The analysis of this section leads to a coherent picture of how energy and quantum constants fit into the time-exponent framework:

- The relativistic and classical definitions of energy agree on the exponent

$$[E] = T^7,$$

independently derived from $E = Mc^2$, $E = FL$, and $E_{\text{grav}} \sim GMm/r$.

- The gravitational product $[G][M]$ occupies the same tier:

$$[G][M] = T^7,$$

linking energy directly to the effective coupling of mass to geometry.

- The reduced Planck constant \hbar lies on the next anchor tier,

$$[\hbar] = T^8,$$

identifying it as a quantum anchor in the global recursive ladder.

- Relativistic momentum resides at the intermediate tier

$$[p] = T^5,$$

between force and energy, consistent with its role as a mediator between dynamical and energetic descriptions.

Together, these results extend the time-recursive hierarchy beyond classical kinematics and gravity into the domains of relativistic energy and quantum scales. No new base units are introduced; instead, energy, momentum, and \hbar find natural positions on the same time-generated ladder that already organizes length, mass, force, and gravitational coupling. The next section applies similar reasoning to the electromagnetic field and the role of electric charge.

6 Electromagnetic Fields and Charge

The time-exponent hierarchy developed so far accommodates kinematics, gravity, and relativistic energy without introducing any new primitive units. We now ask whether classical electromagnetism can be fit into the same structure by assigning a suitable time-exponent to electric charge and its coupling constant.

6.1 Coulomb's Law and the Dimension of Charge

In the electrostatic limit, the force between two point charges q_1 and q_2 at separation r is given by Coulomb's law,

$$F = K \frac{q_1 q_2}{r^2}, \tag{35}$$

where K is the Coulomb coupling constant (for example $K = 1/4\pi\epsilon_0$ in SI units). In the time-exponent framework we assign

$$[q] = T^\sigma, \quad [K] = T^\kappa,$$

with σ and κ initially unknown. Using the previously established exponents

$$[F] = T^4, \quad [r] = T^3, \quad [r^2] = T^6,$$

Eq. (35) implies

$$T^4 = [F] = [K] [q_1][q_2] [r]^{-2} = T^\kappa \cdot T^{2\sigma} \cdot T^{-6} = T^{\kappa+2\sigma-6}. \tag{36}$$

Equating exponents yields the first constraint

$$\kappa + 2\sigma - 6 = 4 \implies \kappa + 2\sigma = 10. \tag{37}$$

The electric field \mathbf{E} generated by a point charge in the static limit satisfies

$$\mathbf{E} \sim K \frac{q}{r^2}, \quad (38)$$

while the force on a test charge is

$$\mathbf{F} = q \mathbf{E}. \quad (39)$$

From Eq. (39) we obtain

$$[E] = \frac{[F]}{[q]} = \frac{T^4}{T^\sigma} = T^{4-\sigma}. \quad (40)$$

On the other hand, Eq. (38) implies

$$[E] = [K][q][r]^{-2} = T^\kappa \cdot T^\sigma \cdot T^{-6} = T^{\kappa+\sigma-6}. \quad (41)$$

Equating Eqs. (40) and (41) gives

$$4 - \sigma = \kappa + \sigma - 6 \implies \kappa + \sigma = 10. \quad (42)$$

Combining Eqs. (37) and (42),

$$\kappa + 2\sigma = 10, \quad (43)$$

$$\kappa + \sigma = 10, \quad (44)$$

we subtract to obtain

$$\sigma = 0, \quad \kappa = 10. \quad (45)$$

Thus electric charge is *dimensionless* in the time-exponent framework,

$$[q] = T^0, \quad (46)$$

and the electromagnetic coupling constant occupies a higher-tier exponent,

$$[K] = T^{10}. \quad (47)$$

With $\sigma = 0$, the electric field and force share the same time exponent,

$$[E] = T^{4-\sigma} = T^4, \quad [F] = T^4. \quad (48)$$

The distinction between “field” and “force” is therefore not dimensional but structural: both live on the T^4 anchor tier, but correspond to different roles in the local four-stage cycle at that tier (field versus force stage).

6.2 Magnetic Field and Electromagnetic Potentials

In relativistic electrodynamics, electric and magnetic fields propagate as waves at the speed of light c . In units where the wave equation reads schematically

$$\square \mathbf{E} = 0, \quad \square \mathbf{B} = 0, \quad c^2 = \frac{1}{\epsilon_0 \mu_0},$$

the magnetic field \mathbf{B} is related to \mathbf{E} by

$$|\mathbf{E}| \sim c |\mathbf{B}| \quad (49)$$

for plane waves. Translating this into dimensions,

$$[E] = [c][B]. \quad (50)$$

Using $[E] = T^4$ and $[c] = [v] = T^2$, we obtain

$$[B] = \frac{[E]}{[c]} = \frac{T^4}{T^2} = T^2. \quad (51)$$

Thus the magnetic field lives on the same tier as velocity and the speed of light:

$$[B] = T^2. \quad (52)$$

This is consistent with the view of \mathbf{B} as an intrinsically “kinematic” field, geometrically linked to rotations and motion of electric field lines.

The electromagnetic four-potential (ϕ, \mathbf{A}) provides a compact relativistic description of the fields. The force on a charge can be derived from the Lagrangian with minimal coupling,

$$p_\mu \rightarrow p_\mu - qA_\mu,$$

so that the spatial vector potential \mathbf{A} must share the same dimension as momentum divided by charge. Because q is dimensionless, this implies

$$[\mathbf{A}] = [p]. \quad (53)$$

From Section 5.5, we have

$$[p] = \frac{[E]}{[c]} = \frac{T^7}{T^2} = T^5, \quad (54)$$

so

$$[\mathbf{A}] = T^5. \quad (55)$$

The scalar potential ϕ appears in the relation between energy and potential energy,

$$E_{\text{pot}} = q\phi.$$

With q dimensionless and $[E_{\text{pot}}] = [E] = T^7$, we obtain

$$[\phi] = T^7. \quad (56)$$

Thus the scalar potential lives on the same tier as energy and the gravitational combination GM , while the vector potential lives on the intermediate momentum tier T^5 .

6.3 Electromagnetic Energy Density

The energy density of the electromagnetic field in vacuum can be written schematically as

$$u \sim \frac{E^2}{K} + \frac{B^2 c^2}{K}. \quad (57)$$

Using the assignments

$$[E] = T^4, \quad [B] = T^2, \quad [c] = T^2, \quad [K] = T^{10},$$

the electric contribution has exponent

$$\left[\frac{E^2}{K} \right] = \frac{T^8}{T^{10}} = T^{-2}, \quad (58)$$

and the magnetic contribution has exponent

$$\left[\frac{B^2 c^2}{K} \right] = \frac{T^4 \cdot T^4}{T^{10}} = T^{-2}. \quad (59)$$

Thus

$$[u] = T^{-2}. \quad (60)$$

On the other hand, energy density is energy per volume. From earlier sections,

$$[E] = T^7, \quad [L] = T^3, \quad [L^3] = T^9,$$

so

$$\left[\frac{E}{L^3} \right] = \frac{T^7}{T^9} = T^{-2}. \quad (61)$$

The electromagnetic expression for u is therefore consistent with the generic definition of energy density in the time-exponent framework.

This consistency check is non-trivial: it confirms that the high-tier placement $[K] = T^{10}$ is compatible with both Coulomb forces and the energy content of electromagnetic fields, without requiring any additional base dimensions.

6.4 Fine-Structure Constant and Dimensionless Couplings

In conventional units, the fine-structure constant α can be written as

$$\alpha = \frac{q^2}{4\pi\epsilon_0\hbar c}, \quad (62)$$

or, in terms of the Coulomb coupling constant K ,

$$\alpha \sim \frac{q^2 K}{\hbar c}. \quad (63)$$

In the time-exponent framework this becomes

$$[\alpha] = \frac{[q]^2[K]}{[\hbar][c]} = \frac{T^0 \cdot T^{10}}{T^8 \cdot T^2} = T^{10-8-2} = T^0. \quad (64)$$

Thus α is dimensionless, as expected.

More importantly, this calculation reveals a structural relationship between the tiers associated with electromagnetic coupling, quantum action, and relativistic kinematics:

$$[K] = T^{10}, \quad [\hbar] = T^8, \quad [c] = T^2.$$

The fact that these exponents add up to a dimensionless combination reflects a compatibility between electromagnetism and quantum mechanics within the same time-recursive ladder; no new primitive is needed to accommodate the fine-structure constant.

7 Discussion and Outlook

We summarize the main dimensional placements obtained in this work and briefly indicate directions for further development.

7.1 Summary of the Time-Exponent Hierarchy

Table 1 collects the key assignments for the lowest tiers of the time-exponent ladder.

Several structural patterns emerge:

- Derived geometric quantities (length and mass) sit at T^3 , just below the force anchor T^4 .

Exponent	Representative quantities
T^0	Electric charge q
T^1	Time t , acceleration a , gravitational acceleration g
T^2	Velocity v , speed of light c , magnetic field B
T^3	Spatial length L , radii r, R , mass M
T^4	Force F , gravitational constant G , electric field E
T^5	Momentum p , vector potential \mathbf{A}
T^7	Energy E , gravitational product GM , scalar potential ϕ
T^8	Reduced Planck constant \hbar (quantum anchor)
T^{10}	Electromagnetic coupling K (Coulomb constant)

Table 1: Representative placements of physical quantities on the recursive time-exponent ladder. Many additional quantities (e.g. energy density, pressure, stress) can be built from these by algebraic combination.

- Dynamical mediators (velocity, magnetic field) occupy T^2 , between the base tier T^1 and the force tier T^4 .
- Momentum and the vector potential share T^5 , mediating between force and energy.
- The energy–curvature tier T^7 hosts both relativistic energy and the gravitational product GM , as well as the scalar potential.
- The reduced Planck constant \hbar lies on the quantum anchor tier T^8 , one step above the energy tier, while the electromagnetic coupling K appears at T^{10} , combining with \hbar and c to form a dimensionless fine-structure constant.

Across kinematics, gravity, relativistic energy, and electromagnetism, all of these quantities are expressed as powers of a single primitive unit of time. No new base dimensions are introduced; instead, the apparent diversity of dimensional types emerges from their placement on a shared recursive ladder of time exponents.

7.2 Conceptual Implications

Interpreting all dimensional quantities as powers of a single temporal primitive recasts several familiar relationships:

- Mass and length share the same exponent T^3 , suggesting that inertial mass is fundamentally geometric rather than an independent dimensional type.
- Energy and gravitational coupling are unified at the exponent T^7 , indicating that the ability of mass to curve the temporal hierarchy is dimensionally tied to its energy content.
- Quantum mechanics enters not as a separate sector but as an additional anchor tier (T^8) in the same recursive hierarchy, with \hbar serving as a quantum anchor above the energy level.
- Electromagnetic interactions fit consistently into this structure once charge is taken to be dimensionless and the Coulomb coupling is placed at T^{10} , leading to correct dimensions for forces, fields, and energy density.

These observations do not by themselves constitute a new dynamical theory of gravity or electromagnetism; rather, they expose a unified dimensional scaffold within which existing laws can be recast. The recursive time-exponent hierarchy acts as an organizing principle for interaction strengths, field roles, and conserved quantities.

7.3 Open Questions and Future Directions

Several avenues for further work are opened by this framework:

- **Curved spacetime and field equations.** The present analysis has focused on weak-field gravity and classical electromagnetism. Extending the time-exponent framework to the full Einstein equations and to non-linear electromagnetic phenomena could clarify how curvature tensors and stress-energy fit into the hierarchy.
- **Field Lagrangians and actions.** The dimensional structure of Lagrangian densities and action integrals provides another testing ground. In particular, understanding how the requirement of a dimensionless path integral weight constrains the exponents of fields and couplings may reveal additional anchor tiers or preferred combinations.
- **Quantum fields and renormalization.** The placement of \hbar at T^8 suggests a natural way to track quantum corrections within the hierarchy. Investigating how running couplings and renormalization-group flows appear when all quantities are expressed as time exponents is a promising direction.
- **Additional interactions.** The strong and weak nuclear forces, as well as possible beyond-Standard-Model sectors, could be analyzed to see whether their effective couplings and characteristic scales select distinct positions on the same ladder.

In summary, the recursive time-dimensional framework presents a unified language in which kinematic, gravitational, relativistic, quantum, and electromagnetic quantities are all organized as powers of a single primitive unit of time. The consistency checks performed here indicate that this language is compatible with standard laws across multiple regimes, while also suggesting new structural links between geometry, energy, and interaction strengths that merit further investigation.

Lastly to anyone reading this far, I want to say thank you to my grade 5 math teacher Mr. Sibbery, 3Blue1Brown for your game changing visualizations of dimensionality, mathologer, numberphile, my parents and my wife. Thank you to the rest of the world also this is the culmination of my lifes work.