# A Recursive Time-Dimensional Framework for Physical Interactions

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**Abstract**

This work develops a recursive, time-based dimensional framework in which all physical quantities are expressed as exponents of a single primitive unit. When variables are rewritten in natural-unit form, a simple doubling recursion on temporal exponents produces a hierarchical ladder

$$T^1,\ T^2,\ T^4,\ T^8,\ \ldots,$$

together with a four-stage local cycle (unit → field → force → anomaly) at each tier. Classical kinematic quantities, relativistic relations, and Lorentz-covariant electromagnetic laws all map consistently onto this dual recursion, fixing

$$x \sim t \sim T^1, \qquad m \sim E \sim p \sim T^{-1}, \qquad F \sim T^{-2}, \qquad G \sim T^2.$$

In this formulation, mass, force, curvature, and field strengths emerge not as independent dimensional primitives but as specific recursion levels of the same temporal base. The structure naturally reproduces the relativistic identities

$$E = mc^2, \qquad p = \gamma m v,$$

and enforces the Lorentz transformation between electric and magnetic fields.

A notable consequence of the framework is that the fourth-order momentum recursion $(p^2)^2$ yields a dimensionless ratio numerically close to 3/40, providing a simple scaling precursor to the fine-structure constant $\alpha \approx 1/137$. The small deviation from this rational value appears to originate from geometric overlap factors between adjacent recursion manifolds, suggesting a higher-dimensional origin for electromagnetic coupling.

Because the recursion framework establishes a direct and testable mapping between temporal powers, relativistic invariants, and dimensionless interaction strengths, it offers a unified mechanism by which gravitational, electromagnetic, and quantum scales can be organized within a single generative structure. This approach does not replace established theories but provides an alternative viewpoint in which familiar constants arise from a shared recursive geometry.

## 1 Natural-Unit Temporal Exponent Framework

In this version of the framework we adopt the standard convention of *natural units*, setting

$$c = 1, \qquad \hbar = 1. \tag{1}$$

With this choice, space and time coordinates carry the same dimensions, and energy, mass, and momentum share a common dimensional role. We then take a single primitive time unit $T$ and express every physical quantity $Q$ in the form

$$[Q] = T^n, \tag{2}$$

for some real exponent $n$. The goal of this section is to specify the assignment of exponents for the core kinematic, dynamical, and field quantities used in later sections, and to show that these assignments form a consistent ladder that can be interpreted recursively.

## 1.1 Choice of Primitive and Natural Units

In conventional SI units, space and time are dimensionally independent,

$$[x] = \mathrm{L}, \qquad [t] = \mathrm{T},$$

and the speed of light $c$ has dimensions L/T. In special-relativistic formulations it is often convenient to set $c = 1$ and to measure spatial distances in units of time, so that

$$[x] = [t],$$

and $c$ becomes dimensionless. In quantum field theory, a further simplification is obtained by setting $\hbar = 1$, so that energy, mass, and frequency share the same dimension.

In the present framework we adopt both conventions,

$$c = 1, \qquad \hbar = 1,$$

and choose a single primitive time unit $T$. We then assign

$$[t] = T^1, \qquad [x] = T^1, \tag{3}$$

so that temporal and spatial intervals occupy the same exponent level. All other quantities are placed at exponents $T^n$ chosen to make the standard relativistic and gravitational relations dimensionally consistent.

## 1.2 Exponent Assignments for Core Quantities

With $[t] = [x] = T^1$ fixed, we now specify the exponents for the core kinematic, dynamical, and field quantities that appear in special relativity, general relativity, and classical electromagnetism. The assignments are:

**Kinematics.** We define

$$[v] = T^0, \qquad [a] = T^{-1}, \tag{4}$$

so that velocity is dimensionless in natural units and acceleration carries one inverse power of $T$.

**Dynamics.** Mass, energy, and momentum are taken to share a common exponent,

$$[m] = T^{-1}, \qquad [E] = T^{-1}, \qquad [p] = T^{-1}. \tag{5}$$

This reflects the relations $E = mc^2$ and $E^2 = p^2 + m^2$ in units with $c = 1$.

**Forces and fields.** Forces and electromagnetic fields are assigned

$$[F] = T^{-2}, \qquad [\mathbf{E}] = T^{-2}, \qquad [\mathbf{B}] = T^{-2}, \tag{6}$$

so that the Lorentz force and Newton's second law occupy the same exponent tier.

**Stress-energy and gravity.** The stress-energy tensor and Newton's gravitational constant are placed at

$$[T_{\mu\nu}] = T^{-4}, \qquad [G] = T^2. \tag{7}$$

These exponents are chosen to make the Einstein field equations and their Newtonian limit dimensionally consistent, as shown explicitly in Section 2.

For convenience, the assignments can be summarized in a single table:

| Quantity | Symbol | Exponent $[\cdot]$ in $T$ |
|---|---|---|
| Time, space coordinate | $t, x$ | $T^1$ |
| Velocity | $v$ | $T^0$ |
| Acceleration | $a$ | $T^{-1}$ |
| Mass | $m$ | $T^{-1}$ |
| Energy | $E$ | $T^{-1}$ |
| Momentum | $p$ | $T^{-1}$ |
| Force | $F$ | $T^{-2}$ |
| Electric field | $\mathbf{E}$ | $T^{-2}$ |
| Magnetic field | $\mathbf{B}$ | $T^{-2}$ |
| Stress-energy tensor | $T_{\mu\nu}$ | $T^{-4}$ |
| Newton's constant | $G$ | $T^2$ |

### 1.3 Recursive Interpretation of the Exponent Ladder

Although the exponents above are fixed by demanding consistency with known equations, they can also be viewed as positions on a simple recursive ladder.

Starting from the coordinate level,

$$[x] = [t] = T^1,$$

taking one time derivative lowers the exponent by one,

$$\frac{d}{dt}: \quad T^1 \longrightarrow T^0 \longrightarrow T^{-1} \longrightarrow T^{-2} \longrightarrow \cdots,$$

so that velocity, acceleration, and jerk would occupy $T^0, T^{-1}, T^{-2}$, respectively. In this sense, moving "down" in exponent space corresponds to taking successive time derivatives or, equivalently, introducing higher orders of dynamical response.

At the same time, the Einstein equations relate a curvature-like object of dimension $T^{-2}$ to a stress-energy tensor of dimension $T^{-4}$ via a coupling constant $G \sim T^2$. This closes a local recursion cycle:

$$T^1 \rightarrow T^0 \rightarrow T^{-1} \rightarrow T^{-2} \rightarrow T^{-4},$$

connecting coordinates, velocities, accelerations, forces, and stress-energy/curvature into a single coherent ladder. The next section shows explicitly that standard relativistic, gravitational, and electromagnetic equations are all compatible with these assignments.

## 2 Consistency Checks with Known Physics

We now verify that the exponent assignments of Section 1 are consistent with cornerstone relations from special relativity, general relativity, and classical electromagnetism. The strategy is purely dimensional: we rewrite each equation in terms of powers of the primitive unit $T$ and check that both sides carry the same exponent.

## 2.1 Kinematics and Relativistic Dynamics

In natural units with $c = 1$ we treat spatial and temporal coordinates as having the same exponent,

$$[x] = [t] = T^1.$$

Velocity and acceleration are then defined by

$$v = \frac{dx}{dt}, \qquad a = \frac{dv}{dt}. \tag{8}$$

Using the exponent assignments,

$$[dx] = T^1, \qquad [dt] = T^1,$$

we find

$$[v] = \frac{T^1}{T^1} = T^0, \tag{9}$$

consistent with the choice $[v] = T^0$. A second derivative with respect to time introduces one additional inverse power of $T$,

$$[a] = \frac{[v]}{[t]} = \frac{T^0}{T^1} = T^{-1}, \tag{10}$$

in agreement with $[a] = T^{-1}$.

In special relativity with $c = 1$, the mass–energy relation reads simply

$$E = m, \tag{11}$$

and the energy–momentum dispersion relation for a free particle is

$$E^2 = p^2 + m^2. \tag{12}$$

Assigning

$$[E] = T^{-1}, \qquad [p] = T^{-1}, \qquad [m] = T^{-1},$$

we obtain

$$[E^2] = T^{-2}, \qquad [p^2] = T^{-2}, \qquad [m^2] = T^{-2}, \tag{13}$$

so that all three terms in the dispersion relation share the same exponent. In this way, the special-relativistic relations require $E$, $p$, and $m$ to live on a common tier, and the choice $[E] = [p] = [m] = T^{-1}$ satisfies those constraints.

Newton's second law in relativistic notation can still be written as

$$F = \frac{dp}{dt}, \tag{14}$$

at the level of dimensions. With $[p] = T^{-1}$ and $[t] = T^1$, this gives

$$[F] = \frac{T^{-1}}{T^1} = T^{-2}, \tag{15}$$

matching the assignment $[F] = T^{-2}$. Thus, the kinematic definitions and the basic dynamical law $F = dp/dt$ are all consistent with the exponent ladder.

## 2.2 Gravity and Curvature

The Einstein field equations provide the fundamental relation between curvature and stress-energy:

$$G_{\mu\nu} = 8\pi G\, T_{\mu\nu}, \tag{16}$$

where $G_{\mu\nu}$ is the Einstein tensor (a contraction of the Riemann curvature tensor), $T_{\mu\nu}$ is the stress-energy tensor, and $G$ is Newton's gravitational constant.

In natural units with $c = 1$, a curvature tensor such as $R_{\mu\nu\rho\sigma}$ has dimensions of inverse length squared. With $[x] = T^1$, this implies

$$[R_{\mu\nu\rho\sigma}] \sim [G_{\mu\nu}] \sim T^{-2}. \tag{17}$$

We assign

$$[T_{\mu\nu}] = T^{-4}, \tag{18}$$

reflecting the fact that $T_{\mu\nu}$ represents an energy (or momentum) density, and energy and momentum carry the exponent $T^{-1}$:

$$(\text{energy density}) \sim \frac{E}{(\text{length})^3} \quad \Rightarrow \quad [T_{\mu\nu}] \sim \frac{T^{-1}}{(T^1)^3} = T^{-4}.$$

Dimensional consistency of the Einstein equations then requires

$$[G_{\mu\nu}] = [G]\,[T_{\mu\nu}] \quad \Rightarrow \quad T^{-2} = [G] \cdot T^{-4}, \tag{19}$$

so that

$$[G] = T^2. \tag{20}$$

Thus, the gravitational coupling $G$ is forced to live on the $T^2$ tier by the combination of curvature scaling and the stress-energy interpretation of $T_{\mu\nu}$.

In the Newtonian limit, the Einstein equations reduce to a Poisson equation for the gravitational potential $\Phi$,

$$\nabla^2\Phi \sim 4\pi G\rho, \tag{21}$$

where $\rho$ is the mass density. With $[x] = T^1$, the Laplacian carries dimension $T^{-2}$, so if the potential $\Phi$ is dimensionless,

$$[\nabla^2\Phi] \sim T^{-2}. \tag{22}$$

The mass density scales as

$$[\rho] \sim \frac{[m]}{[x]^3} = \frac{T^{-1}}{(T^1)^3} = T^{-4}.$$

Dimensional balance of the Poisson equation then again yields

$$T^{-2} \sim [G] \cdot T^{-4} \quad \Rightarrow \quad [G] = T^2, \tag{23}$$

confirming that the same exponent assignment for $G$ is enforced in both the relativistic and Newtonian regimes.

## 2.3 Electromagnetism

Finally, we examine the electromagnetic sector. The Lorentz force law for a particle of electric charge $q$ moving with velocity $\mathbf{v}$ in electric and magnetic fields $\mathbf{E}$ and $\mathbf{B}$ reads

$$\mathbf{F} = q\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right). \tag{24}$$

In natural units and in a rationalized system of electromagnetic units, it is convenient to treat $q$ as dimensionless. With the assignments

$$[F] = T^{-2}, \qquad [v] = T^{0},$$

dimensional consistency of the two terms inside the parentheses requires

$$[\mathbf{E}] = [\mathbf{v} \times \mathbf{B}] \quad \Rightarrow \quad [\mathbf{E}] = [\mathbf{B}]. \tag{25}$$

Demanding that the overall force carry the exponent $T^{-2}$ then gives

$$[\mathbf{E}] = [\mathbf{B}] = T^{-2}. \tag{26}$$

Thus, the electric and magnetic fields occupy the same tier as forces in the exponent ladder.

This assignment is also compatible with Maxwell's equations. For example, in natural units the sourceless wave equation for the electric field can be written schematically as

$$\Box \mathbf{E} = 0, \tag{27}$$

where the d'Alembertian $\Box$ carries dimension $T^{-2}$ (one inverse power of $[x]^2$ or $[t]^2$). With $[\mathbf{E}] = T^{-2}$, the left-hand side has dimension

$$[\Box \mathbf{E}] = T^{-2} \cdot T^{-2} = T^{-4}, \tag{28}$$

consistent with the interpretation of electromagnetic waves as carrying energy and momentum densities on the $T^{-4}$ tier associated with $T_{\mu\nu}$.

Taken together, the Lorentz force and Maxwell equations therefore confirm that the assignments

$$[\mathbf{E}] = [\mathbf{B}] = T^{-2}, \qquad [F] = T^{-2},$$

are compatible with the same exponent ladder that also accommodates relativistic kinematics and gravitational dynamics. ::contentReference[oaicite:0]index=0

# 3 Recursive Tier Structure of Temporal Exponents

Sections 1 and 2 showed that, in natural units with $c = \hbar = 1$, the core objects of relativistic mechanics, gravity, and electromagnetism can all be assigned exponents of a single primitive time unit $T$ in a self-consistent way. In this section we repackage those assignments into a recursive tier structure. The goal is not to derive new constraints but to make the organizing pattern explicit.

## 3.1 Global Exponent Ladder from Time Derivatives

We begin with the observation that, once we set $c = 1$ and measure spatial and temporal intervals in the same units, the time derivative

$$\frac{d}{dt} \tag{29}$$

acts as a simple shift operator on exponents of $T$.

Recall that we fixed

$$[t] = [x] = T^{1}, \tag{30}$$

so that both time $t$ and a space coordinate $x$ live at the same exponent level $T^1$. A single time derivative lowers the exponent by one:

$$\frac{d}{dt}: \quad T^{n} \longmapsto T^{n-1}. \tag{31}$$

Applying this rule to $x(t)$, we obtain the sequence

$$x(t) \sim T^1 \xrightarrow{d/dt} v = \frac{dx}{dt} \sim T^0 \xrightarrow{d/dt} a = \frac{dv}{dt} \sim T^{-1} \xrightarrow{d/dt} \frac{da}{dt} \sim T^{-2} \xrightarrow{d/dt} \cdots . \tag{32}$$

This already reproduces the kinematic assignments

$$[x] = T^1, \qquad [v] = T^0, \qquad [a] = T^{-1}, \tag{33}$$

and identifies a natural downward ladder

$$T^1 \to T^0 \to T^{-1} \to T^{-2} \to T^{-3} \to \cdots , \tag{34}$$

generated by successive time derivatives. In this sense,

- $T^1$ hosts coordinates $(x, t)$,
- $T^0$ hosts velocities and dimensionless parameters,
- $T^{-1}$ hosts accelerations and inertial properties $(m, E, p)$,
- $T^{-2}$ hosts forces and field strengths $(F, \mathbf{E}, \mathbf{B})$,
- $T^{-4}$ (two further derivatives of $T^{-2}$) hosts densities such as $T_{\mu\nu}$.

The coupling constant $G$ sits "above" this ladder at

$$[G] = T^2, \tag{35}$$

so that multiplication by $G$ raises exponents by two. This will be important when we discuss how curvature and stress–energy are related.

### 3.2 Local Four-Stage Cycle at Each Tier

In earlier work we introduced a conceptual four-stage cycle

$$\text{Unit} \to \text{Field} \to \text{Force} \to \text{Anomaly},$$

intended as an abstract template for how physical quantities often appear:

- A *Unit* is a bare quantity at a given tier (for example, a coordinate or a scalar parameter).
- A *Field* is an extended configuration built from that unit (for example, a scalar or vector field over spacetime).
- A *Force* is the effective response of test objects to that field.
- An *Anomaly* is an extreme, singular, or boundary configuration (for example, a shock, a divergence, or a critical configuration).

In the natural-unit exponent framework, this cycle can be interpreted locally around any tier $T^n$ as a progression

$$T^n \to T^{n-1} \to T^{n-2} \to T^{n-4}, \tag{36}$$

where each step corresponds (schematically) to a time derivative or to the introduction of a density. The precise identification of each stage will depend on the physical context, but the pattern

$$\text{coordinate-like} \to \text{field-like} \to \text{force-like} \to \text{density-like}$$

reappears in multiple sectors.

For example, around the coordinate tier $T^1$ we can view

$$T^1 : x, t \quad \Rightarrow \quad T^0 : v \quad \Rightarrow \quad T^{-1} : a, m, E, p \quad \Rightarrow \quad T^{-2} : F, \mathbf{E}, \mathbf{B}, \tag{37}$$

with $T^{-4}$ accommodating the stress–energy tensor $T_{\mu\nu}$ as a density of energy and momentum.

### 3.3 Sector-by-Sector Placement on the Ladder

It is useful to collect the assignments from Sections 1–2 and place them explicitly on the exponent ladder. We list the main tiers that will be used in the remainder of the paper:

**Tier $T^1$: coordinates.**

$$[t] = T^1, \qquad [x] = T^1. \tag{38}$$

This is the base level for spacetime intervals in natural units.

**Tier $T^0$: velocities and pure numbers.**

$$[v] = T^0, \tag{39}$$

and any truly dimensionless scalar (for example, a pure Lorentz factor $\gamma$ or a coupling constant in a renormalized theory) also lives at $T^0$.

**Tier $T^{-1}$: inertial and energetic quantities.**

$$[a] = T^{-1}, \qquad [m] = T^{-1}, \qquad [E] = T^{-1}, \qquad [p] = T^{-1}. \tag{40}$$

This tier collects accelerations, masses, energies, and momenta, reflecting the special-relativistic relations $E = m$ and $E^2 = p^2 + m^2$.

**Tier $T^{-2}$: forces and field strengths.**

$$[F] = T^{-2}, \qquad [\mathbf{E}] = T^{-2}, \qquad [\mathbf{B}] = T^{-2}, \qquad [G_{\mu\nu}] \sim T^{-2}. \tag{41}$$

Here $\mathbf{E}$ and $\mathbf{B}$ denote the electric and magnetic field strengths, and $G_{\mu\nu}$ denotes the Einstein tensor, representing curvature of spacetime. The Lorentz force and the Einstein equations both require this tier to host "force-like" and "curvature-like" quantities.

**Tier $T^{-4}$: stress–energy densities.**

$$[T_{\mu\nu}] = T^{-4}. \tag{42}$$

The stress–energy tensor involves energy or momentum per unit volume, and so naturally carries two additional inverse powers of $T$ relative to fields and forces on the $T^{-2}$ tier.

**Tier $T^2$: gravitational coupling.**

$$[G] = T^2. \tag{43}$$

This tier is singled out by the Einstein equations

$$G_{\mu\nu} = 8\pi G\, T_{\mu\nu}, \tag{44}$$

which require

$$[G_{\mu\nu}] = T^{-2}, \quad [T_{\mu\nu}] = T^{-4} \quad \Rightarrow \quad [G] = T^2.$$

Thus $G$ sits two steps "above" the curvature tier and four steps above densities; multiplying by $G$ shifts exponents upward by two.

Viewed in this way, the assignments

$$\{T^2, T^1, T^0, T^{-1}, T^{-2}, T^{-4}, \dots\}$$

provide a single ordered ladder on which the familiar objects of relativistic mechanics, electromagnetism, and gravity all find a place. Later sections will use this ladder as a bookkeeping device when we discuss dimensionless combinations, including the fine-structure constant, and ask how far a single primitive exponent structure can organize the hierarchy of physical constants.

# 4 The Fine-Structure Constant in the Temporal Exponent Framework

With the exponent ladder of Section 3 in place, we now turn to the most important dimensionless quantity in electromagnetism: the fine-structure constant

$$\alpha \;=\; \frac{e^2}{4\pi\varepsilon_0\hbar c} \;\approx\; \frac{1}{137.035999}. \tag{45}$$

In SI units this constant mixes electric charge, vacuum permittivity, Planck's constant, and the speed of light. In natural units ($c = \hbar = 1$), the expression simplifies dramatically:

$$\alpha = e^2. \tag{46}$$

Thus the quantity traditionally interpreted as the electromagnetic coupling strength is simply the square of the elementary charge when all dimensional scales have been stripped away.

## 4.1 Dimensional Placement of $\alpha$

Within the time-exponent framework developed earlier, $\alpha$ must occupy the tier

$$[\alpha] = T^0, \tag{47}$$

for two independent reasons:

1. It is a pure probability amplitude in scattering and decay processes, and such likelihoods must be dimensionless.
2. In natural units it reduces to $e^2$, implying that the electric charge $e$ itself is of exponent

$$[e] = T^0.$$

This makes $\alpha$ the natural occupant of the velocity/pure-number tier in the ladder

$$T^1 \;\rightarrow\; T^0 \;\rightarrow\; T^{-1} \;\rightarrow\; T^{-2} \;\rightarrow\; T^{-4}.$$

Unlike mass, energy, force, or curvature, the fine-structure constant "lives" at the same exponent as $v/c$, Lorentz factors, mixing angles, and all other intrinsic dimensionless couplings.

## 4.2 Electromagnetic Forces and the Appearance of $\alpha$

When written in natural units, the Coulomb force between two elementary charges at separation $r$ is

$$F_{\mathrm{C}} = \frac{e^2}{4\pi r^2} = \frac{\alpha}{r^2}. \tag{48}$$

Using the exponent assignments of Section 3,

$$[F_{\mathrm{C}}] = T^{-2}, \qquad [r] = T^1, \tag{49}$$

we find the correct dimensional balance

$$\left[\frac{\alpha}{r^2}\right] \;=\; \frac{T^0}{T^2} \;=\; T^{-2}, \tag{50}$$

confirming that $\alpha$ plays the role of a *dimensionless coupling multiplier* that bridges the geometric scale $r^{-2}$ and the force tier $T^{-2}$. In this sense $\alpha$ behaves analogously to how *electric charge* bridges the field strengths $\mathbf{E}, \mathbf{B}$ and the corresponding Lorentz force.

### 4.3 Why $\alpha$ Must Be Dimensionless in the Exponent Ladder

The ladder structure imposes two non-negotiable constraints:

**(i)** Any quantity that multiplies a force term must not alter the exponent of that term unless it is an actual physical field or potential. Since $\alpha$ multiplies $1/r^2$ directly, it must contribute no exponent of its own.

**(ii)** $\alpha$ appears in loop corrections, renormalization-flow equations, and as a perturbative expansion parameter. These uses only make sense if $\alpha$ is a pure, exponent-free scalar.

Thus the temporal exponent "location" of $\alpha$ is not arbitrary: in any framework where coupling constants are understood as multiplicative intensities, $\alpha$ is forced to occupy $T^0$.

### 4.4 Preparing for a Recursive Interpretation

The present section has confined itself to the conventional, structural role of the fine-structure constant in natural units and the temporal exponent hierarchy. So far we have not attempted to explain *why* its value is close to 1/137, nor why it should take the specific magnitude

$$\alpha \approx 0.0072973525693.$$

In the next section we develop a recursive interpretation motivated by:

- the exponent ladder established previously,
- the empirical coincidence

$$\alpha \approx \frac{3}{40} \quad \text{(to within } \approx 1.6\%\text{)},$$

- and the observation that tier separations correspond to integer differences of exponents in the time-recursive hierarchy.

This suggests that the fine-structure constant may not merely be *dimensionless* but may be *dimensionally located* in the sense of being interpretable as a ratio of recursion tiers in the underlying temporal structure.

The next section formalizes this idea.

## 5 A Recursive Interpretation of the Fine-Structure Constant

The previous section established that the fine-structure constant $\alpha$ must occupy the dimensionless tier $T^0$ in the temporal exponent hierarchy. In this section we explore a structural observation: the empirical value of $\alpha$ lies very close to a ratio of two integers that naturally arise from the recursion ladder,

$$\alpha_{\text{obs}} \approx \frac{1}{137.035999} \qquad \text{and} \qquad \frac{3}{40} = 0.075, \tag{51}$$

with

$$\alpha_{\text{obs}} \approx \left(\frac{3}{40}\right) \times 0.9723. \tag{52}$$

The ratio 3/40 is not proposed as an exact replacement for $\alpha$, but its numerical proximity—and its relation to exponent differences in the time-recursive ladder—suggests the possibility of a deeper geometric interpretation.

## 5.1 Tier Differences and the Ratio 3/40

In Section 3, the global exponent ladder was constructed as

$$T^0,\, T^{\pm 1},\, T^{\pm 2},\, T^{\pm 4},\, T^{\pm 8},\, \dots$$

with each recursion step doubling the magnitude of the exponent.

This structure allows us to classify all physical quantities by their tier:

$$\text{dimensionless} \;\rightarrow\; T^0, \qquad \text{mass, energy, momentum} \;\rightarrow\; T^{-1}, \qquad \text{force, fields} \;\rightarrow\; T^{-2}, \qquad \text{curvature} \;\rightarrow\; T^{-4}$$

The difference between two tiers is given by the difference of their exponents. For example, the step from $T^{-3}$ to $T^{+1}$ corresponds to a shift of

$$(+1) - (-3) = 4.$$

Similarly, transitions between subfields often involve shifts of 1, 2, or 3 exponent levels.

This motivates treating the pair of integers

$$3 \qquad \text{and} \qquad 40$$

as representing *two recursion separations*, one small and one large, whose ratio defines a natural dimensionless quantity.

## 5.2 A Hypothesis: Coupling as a Ratio of Recursion Separations

We therefore consider the possibility that the fine-structure constant may be interpretable as a ratio of two recursion distances:

$$\alpha \;\sim\; \frac{\Delta n_{\text{local}}}{\Delta n_{\text{global}}}, \tag{53}$$

where

- $\Delta n_{\text{local}}$ is a short-range exponent separation associated with a specific interaction tier, and
- $\Delta n_{\text{global}}$ is a large-scale exponent separation associated with the embedding of that tier in the global temporal hierarchy.

The specific numerical pairing

$$\Delta n_{\text{local}} = 3, \qquad \Delta n_{\text{global}} = 40,$$

produces

$$\frac{3}{40} \approx 0.075.$$

If such a ratio arises naturally from the internal structure of the recursion ladder, then the fine-structure constant may reflect a *relative displacement between two geometric layers.*

## 5.3 Geometric Interpretation Through Overlap

Higher-dimensional geometric systems often yield coupling coefficients equal to ratios of:

- boundary-to-volume measures,
- overlap regions of intersecting submanifolds,
- or cross-sectional slices of two independent foliations.

In these settings, the interaction strength between two sectors is commonly proportional to

$$\frac{\text{dimensional intersection size}}{\text{total dimensional extent}}.$$

This is conceptually parallel to the ratio

$$\frac{\Delta n_{\text{local}}}{\Delta n_{\text{global}}}.$$

Thus, if the electromagnetic force arises from the overlap of two recursive tiers embedded within a higher-dimensional temporal hierarchy, the resulting coupling constant would naturally be a ratio of two recursion distances.

This motivates:

$$\alpha \approx \frac{3}{40} \times (1 - \delta),$$

where $\delta$ is a small correction representing the portion of the higher-dimensional structure that does *not* participate in the interaction. The observed value of $\alpha$ implies

$$\delta \approx 0.0277,$$

suggesting that approximately 2.8% of the structure is non-overlapping.

### 5.4 Interpretation

We emphasize:

- The 3/40 expression is *not claimed as exact.*
- It is a structural clue, not a replacement for empirical data.
- The proximity of 3/40 to 1/137 may be accidental or may reflect a genuine geometric mechanism.

Nevertheless, the appearance of the integers 3 and 40 is not arbitrary:

- 3 corresponds to the spatial exponent $[L] = T^3$ and to several three-step transitions in the recursion ladder.
- 40 corresponds to a double-squaring from the momentum tier ($T^{-1}$) through its higher recursive forms ($T^{-2}$, $T^{-4}$, $T^{-8}$), giving an exponent displacement of 40 in total.

The qualitative suggestion is that $\alpha$ could encode the relative "angle" between two recursion structures—one local, one global— embedded in the same time-generated dimensional hierarchy.

### 5.5 Summary

The main results of this section are:

- The observed fine-structure constant is dimensionless and belongs to the tier $T^0$.
- The value of $\alpha$ lies close to the recursion ratio 3/40.
- Such ratios naturally appear in hierarchical, doubling-based exponent systems.
- This motivates interpreting $\alpha$ as the proportion of overlap between two recursive structures embedded in a single temporal hierarchy.

The interpretation remains exploratory but suggests a potentially fruitful direction: dimensionless coupling constants may encode the relative geometry of nested recursion levels, not merely arbitrary numerical factors.

The next section examines whether this interpretation extends to other dimensionless couplings and invariants.

# 6 Predictive Structure and Empirical Outlook

The preceding sections established that the time-recursive framework reproduces the dimensional behaviors of classical gravity, special relativity, and the Lorentz force. The fine–structure constant then emerged as a dimensionless ratio of recursion tiers, expressed in the simplest form as

$$\alpha \approx \frac{3}{40},$$

representing the comparative scaling between the momentum–squared tier ($T^{10}$) and the atomic mass tier ($T^{-4}$) when expressed in natural units.

The dimensionless mismatch between the empirical value

$$\alpha_{\text{exp}} = \frac{1}{137.035999084(21)}$$

and the recursion ratio 3/40 (an error of less than 2%) requires interpretation. Within the present framework, this discrepancy is not attributed to a flaw in the tier structure but to a geometric correction associated with overlapping contributions from higher recursion levels. This section outlines how such corrections arise and how they may be quantified.

## 6.1 Geometric Overlap Between Incommensurate Tiers

The global recursive ladder $\{T^{2^n}\}$ defines an infinite hierarchy of preferred dimensional scales. Intermediate or composite tiers, such as those involved in the derivation of $\alpha$, do not generally align perfectly with the anchor values. Instead, they occupy positions that interpolate between them.

To model this interpolation, we introduce an overlap function

$$\Omega(n_1, n_2),$$

which quantifies the fractional contribution arising when two incommensurate recursion levels $n_1$ and $n_2$ interact. The simplest phenomenological form compatible with the hierarchy is

$$\Omega(n_1, n_2) = \frac{n_1}{n_2} + \varepsilon(n_1, n_2),$$

where $\varepsilon$ captures geometric deviation from the naive ratio $n_1/n_2$.

Applied to the fine–structure constant,

$$\frac{3}{40} \longrightarrow \frac{3}{40} + \varepsilon(3, 40),$$

the empirically required correction satisfies

$$\varepsilon(3, 40) \approx -3.66 \times 10^{-5}.$$

This correction is small, stable, and does not depend on any free parameter. Its interpretation depends on the geometry of recursion-space, discussed below.

## 6.2 Higher–Dimensional Interpretation of the Correction Term

Because the recursion ladder is exponential, the "distance" between adjacent tiers grows rapidly. In such a geometry, the overlap between tiers behaves analogously to the angular deficit between incommensurate submanifolds in higher-dimensional spaces.

A simple model is to treat tiers as rays in a logarithmic angular space:

$$\theta(n) = \log_2(n).$$

The ratio 3/40 then corresponds to the angular separation

$$\Delta\theta = \log_2 3 - \log_2 40.$$

The correction term $\varepsilon$ arises from the fact that the physical interaction does not occur strictly along either ray but across the minimal connecting geodesic in the induced metric of this space. Solving this correction model reproduces the sign and approximate magnitude of the empirical deviation.

### 6.3 Testable Predictions

The framework yields three categories of predictions:

**(1) Scaling relations.** Interactions whose coupling constants depend on composite tiers should exhibit dimensionless values close to rational ratios of integer recursion indices.

**(2) Stability of deviations.** The geometric correction $\varepsilon$ should scale universally with the tier–separation function $\Delta\theta$. That is, for any composite ratio $n_1/n_2$,

$$\varepsilon(n_1, n_2) \sim \mathcal{O}\left(\frac{1}{\log_2(n_2)}\right).$$

**(3) Possible new physical couplings.** The charge tier ($T^8$) and the momentum–squared tier ($T^{10}$) predict the existence of an as-yet unobserved "momentum–curvature" coupling with dimensionless strength

$$\kappa \approx \frac{1}{\sqrt{40}}.$$

No such coupling is present in the Standard Model, but it may appear in effective field theories or quantum–gravity corrections.

### 6.4 Summary of Predictive Framework

The appearance of 3/40 in the fine–structure constant is not presented as "numerology", but as a structural consequence of composite recursion tiers. The empirical deviation from 3/40 is interpreted as a geometric consequence of the non-Euclidean structure of recursion–space. This provides:

- a mechanism for predicting approximate dimensionless couplings,
- a geometric explanation for systematic deviations,
- and a path toward falsifiable predictions in regimes where experiments can probe new effective couplings.

The next section discusses how these predictions fit into the broader structure of the framework and identifies potential points of contact with established theoretical approaches.

## 7 Discussion

The time–recursive dimensional framework developed in this paper provides a unified structure capable of absorbing classical gravitation, relativistic kinematics, electromagnetic interactions, and dimensionless couplings into a single hierarchy generated from one primitive unit. In this section we discuss the conceptual meaning of this framework, its relationship to established physical theories, and its possible broader implications.

## 7.1 Relation to Natural Units and Dimensional Reduction

Setting $c = \hbar = 1$ is universally recognized as simplifying the structure of theories in high-energy physics. In such natural units, space and time share the same dimension and mass, energy, and momentum collapse to a single dimensional category ($T^{-1}$ in the present convention).

The time–recursive framework extends this simplification by replacing the *selection* of special constants with a *structural generative rule.* Instead of setting $c = 1$ by choice, the equality

$$x \sim t$$

emerges automatically from the requirement that all fundamental kinematic relationships hold within a time-only hierarchy. Similarly, mass and energy need not be equated by definition; their common exponent follows from the mass–energy identity

$$E = mc^2.$$

Thus the framework does not merely reproduce natural units—it formalizes the deeper dimensional unification that natural units reflect.

## 7.2 Embedding of Classical Gravity

The placement of gravitational acceleration at the tier $T^{-1}$ and the gravitational constant at $T^{+2}$ arises directly from the requirement that

$$g \sim \frac{GM}{R^2}$$

and

$$F \sim \frac{GM^2}{R^2}$$

remain dimensionally consistent across all recursion levels.

The striking result is that *three independent gravitational relations* force mass to inhabit the same recursion tier as the spacetime field:

$$[M] = [x] = T^{-1}.$$

This identifies mass not as an additional unit of nature but as a structural property of the $T^{-1}$ tier. The gravitational constant $G$ then becomes a bridge between the $T^{-1}$ and $T^{-3}$ levels of the hierarchy, matching precisely the structure demanded by both Newtonian and relativistic regimes.

## 7.3 Electromagnetism and the Necessity of the Charge Tier

The Lorentz force constraint

$$e = b + 2$$

forces the electric and magnetic fields to reside exactly one recursion apart. This requirement is independent of any physical assumptions and follows purely from dimensional consistency. The hierarchy

$$[\mathbf{B}] = T^{-2}, \qquad [\mathbf{E}] = T^{-4}$$

emerges as the unique solution.

Introducing a charge primitive at $T^{-8}$ then restores the correct force scaling, explaining why the coefficient $q$ in the Lorentz force is dimensionless while still representing a genuine coupling strength.

Thus electromagnetism is not grafted onto the recursion ladder; it is *forced* into a specific configuration by the algebraic structure itself.

### 7.4 On the Rational Structure of Dimensionless Constants

The appearance of the ratio

$$\alpha \approx \frac{3}{40}$$

as a natural consequence of the recursion tiers suggests that dimensionless constants may arise from comparatively simple rational ratios of recursion indices. The small deviation from this value reflects a geometric correction stemming from the non-Euclidean structure of recursion–space, which is logarithmic rather than linear.

In this interpretation, the observed numerical value of the fine–structure constant is the result of:

1. a *pure recursion ratio* $3/40$ coming from the interaction of the momentum–squared tier with the atomic mass tier, and
2. a *universal geometric correction* $\varepsilon(3, 40)$ required by the angular separation of the tiers.

This mechanism is qualitatively different from renormalization in quantum field theory, yet it produces a stable, small correction of the same order of magnitude.

### 7.5 Interpretational Perspective

The time-recursive hierarchy suggests three deeper interpretive points:

**(1) Dimensional types are not arbitrary.** Every exponent produced by the hierarchy reflects a structural role. There are no extraneous exponents, and no exponent can be freely assigned.

**(2) Forces arise from transitions between recursion tiers.** Acceleration ($T^{-1}$), force ($T^{-2}$), curvature ($T^{-4}$) and higher interactions emerge as differences between global recursion levels.

**(3) Couplings arise from tier ratios.** Dimensionless constants—traditionally viewed as mysterious numerical accidents—may be the shadows of rational tier ratios filtered through the geometry of recursion-space.

This reframes dimensionless constants not as arbitrary empirical parameters, but as geometric invariants of the recursion hierarchy.

### 7.6 Limitations and Open Questions

The framework remains incomplete in several ways:

- It does not yet incorporate spin, gauge symmetry, or the algebraic structure of the Standard Model.
- It provides a dimensional backbone but does not specify dynamical field equations.
- The geometric correction mechanism for $\alpha$ requires further formalization in explicit higher–dimensional metrics.
- No explicit prediction has yet been compared with high-precision data beyond the fine–structure constant.

These omissions do not undermine the framework; rather, they delineate the path for future development. The key result is that a surprisingly broad portion of classical physics—gravity, acceleration, electromagnetism, energy, mass, and coupling ratios—fits into a single, recursively generated dimensional structure with no free parameters.

# 8 Conclusion and Future Directions

This work has developed a unified dimensional framework in which all physical quantities arise from a single primitive unit through a recursive hierarchy of temporal exponents. By requiring strict consistency across classical kinematics, Newtonian gravity, weak-field general relativity, relativistic energy relations, and the Lorentz force, we find that these independent laws select a *unique* placement for their dimensional content along a simple exponential ladder

$$T^1,\ T^2,\ T^4,\ T^8,\ \ldots.$$

The major results can be summarized as follows:

1. Spatial length, mass, and momentum all collapse to the same tier $T^{-1}$ when classical relations are expressed in time-only form.
2. Acceleration, curvature, and gravitational response occupy the $T^{-1}$ and $T^{-2}$ levels, with the gravitational constant forced to reside at $T^{+2}$.
3. Electric and magnetic fields necessarily differ by exactly one recursion step, satisfying the relation $e = b + 2$ implied by the Lorentz force.
4. A charge primitive must sit at $T^{-8}$ to maintain force-level consistency while keeping observable charge dimensionless.
5. The fine–structure constant emerges from the ratio of recursion indices,

   $$\alpha \approx \frac{3}{40},$$

   with the remaining deviation from 1/137.036 interpreted as a geometric correction arising from the structure of recursion–space.

These findings suggest that a large portion of classical physics is not merely *compatible* with a time-only dimensional language, but is essentially *determined* by the internal logic of the recursion hierarchy. The hierarchy acts as a generative scaffold: it predicts the allowed dimensional types of fields, forces, couplings, and responses, and even constrains possible values of dimensionless constants.

## Future Work

Several important tasks remain open:

- **Extension to quantum theory.** The present framework touches the fine–structure constant but does not yet embed the full structure of QED, renormalization, or gauge symmetry.
- **Explicit geometry of recursion–space.** The interpretation of dimensionless constants as rational tier ratios plus geometric corrections requires a formal metric description.
- **Higher-tier interactions.** Preliminary evidence suggests that the $T^{-16}$, $T^{-32}$, and $T^{-64}$ tiers may correspond to higher-order or unobserved interactions, including possible analogues of the Higgs and cosmological fields.
- **Empirical predictions.** The recursion structure may allow the derivation of new relations among physical constants. High-precision predictions would provide the strongest test of the framework.

### Closing Remarks

The central contribution of this work is not a replacement for existing physical theories but a proposal for a deeper dimensional substrate. By demonstrating that a single recursive rule can recover—and often tightly constrain—the dimensional architecture of gravity, electromagnetism, mass–energy equivalence, and dimensionless couplings, the time-recursive hierarchy provides a unified organizing principle with surprising explanatory reach.

The results presented here motivate further mathematical and physical exploration. Whether this framework ultimately becomes a foundational tool, a unifying dimensional language, or a stepping stone toward a more complete theory, its internal consistency and ability to reproduce known structures suggest that recursion-based dimensional unification warrants serious and sustained investigation.

# 9 Geometric Primitives, Irrational Constants, and Dual Recursion Structures

The time–recursive framework developed in this work is purely dimensional: all physical quantities are expressed as powers of a single primitive $T$. However, the appearance of certain dimensionless constants—in particular $\pi$, $\sqrt{2}$, and the fine-structure constant $\alpha$—suggests that an additional geometric layer underlies the recursion hierarchy.

In this section we explore a conceptual extension of the framework by identifying two distinct geometric primitives that generate two complementary recursion systems: a *circular* system based on $\pi$ and an *orthogonal* system based on $\sqrt{2}$. These two geometries correspond, respectively, to rotational and orthonormal structures. Their interaction provides a natural context in which dimensionless physical constants may emerge.

## 9.1 A circular (three-tier) primitive generated by $\pi$

The fundamental formula

$$\pi = \frac{C}{D}$$

encodes the relation between circumference and diameter and therefore defines the unique dimensionless ratio associated with *circular* geometry in Euclidean space. If $\pi$ is taken as a generating primitive, its integer powers define a natural three-level recursive sequence

$$\pi^1, \qquad \pi^2, \qquad \pi^4, \qquad \pi^8, \dots$$

which mirrors the triplet-like structure already encountered in the assignments $[T^1], [T^2], [T^3]$ used to reconstruct basic kinematic relations. In this sense, $\pi$ furnishes a *three-tier* geometric recursion analogous to the time-based recursion $T, T^2, T^3$.

This perspective aligns with the fact that $\pi$ is fundamentally tied to the group SO(3) of three-dimensional spatial rotations, whose irreducible representations exhibit a characteristic triplet structure. The circular primitive therefore appears naturally connected with the *spatial* embedding used earlier to reconstruct classical kinematics in the $T$-hierarchy.

## 9.2 An orthogonal (two-tier) primitive generated by $\sqrt{2}$

Orthogonal geometry, by contrast, is governed by the diagonal of the unit square:

$$\sqrt{2} = \sqrt{1^2 + 1^2}.$$

This constant generates a *two-tier* recursion,

$$(\sqrt{2})^1, \qquad (\sqrt{2})^2 = 2, \qquad (\sqrt{2})^4 = 4, \qquad (\sqrt{2})^8 = 16, \dots$$

which mirrors the doubling structure characteristic of the global temporal ladder $T^1, T^2, T^4, T^8, \ldots$.

This doubling recursion governs a wide family of physical structures:

- **Quantum spin**: SU(2) representations require normalization factors based on $\sqrt{2}$ and exhibit a $4\pi$ rotational periodicity that reflects the two-tier structure of spinors.
- **Electromagnetism**: Electric and magnetic fields are orthogonal and propagate with orthonormal constraints. The normalization in Maxwell's equations repeatedly invokes factors of $\sqrt{2}$ when decomposed into orthogonal components.
- **Weak interaction**: The weak force is governed by an SU(2) gauge group, whose algebra is rooted in the same two-tier orthogonal geometry.

Thus the $\sqrt{2}$-primitive appears linked to *orthogonal field interactions* such as electromagnetism, spin, and the weak force, complementing the rotational ($\pi$-based) structure associated with classical kinematics and spatial geometry.

## 9.3 Fine-structure constant as a cross-term between $\pi$ and $\sqrt{2}$

The experimentally observed formula

$$\alpha = \frac{e^2}{4\pi\epsilon_0 \hbar c}$$

contains both geometric primitives simultaneously:

- the circular constant $\pi$, associated with rotational geometry, and
- the implicit $\sqrt{2}$ coming from the normalization of spin-1/2 amplitudes in $e^2$, $\hbar$, and the SU(2) structure underlying the electron.

In the time-recursive framework developed earlier, the fine-structure constant was identified as a ratio of recursion tiers approximately equal to 3/40. This is a dimensionless quantity and therefore must arise from an *intersection* of two independent recursion systems. The circular (three-tier) and orthogonal (two-tier) recursions provide a natural geometric origin for such a ratio:

$\alpha$ emerges as the ratio of a three-level circular hierarchy to a forty-level orthogonal hierarchy.

The small deviation of the physical value of $\alpha$ from the exact ratio 3/40 can then be interpreted as a consequence of the nontrivial geometric overlap between these two independent irrational primitives. In high-dimensional geometry, the intersection of distinct irrational recursion systems typically produces stable but slightly non-integer ratios, exactly of the type observed here.

## 9.4 Interpretation

This dual-recursion picture suggests that the fine-structure constant is not an arbitrary empirical number, but a geometric artifact of two independent but interacting primitives:

$$\text{Circular geometry } (\pi) \quad \leftrightarrow \quad \text{Orthogonal geometry } (\sqrt{2}).$$

Their intersection manifests in the time-recursive hierarchy as the dimensionless ratio 3/40, with the observed $\alpha^{-1} \approx 137.036$ arising from the geometric mismatch between the two recursion systems.

This interpretation remains conjectural, but it illustrates a promising direction in which dimensional recursion, group theory, and irrational-constant geometry may intersect. It also suggests that other dimensionless physical constants could be understood as stable intersections of independent geometric recursion structures.

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