Finding the Physical Unit 1: Balmer Normalization for a Recursive Phase–Transport Model of Physics

Patrick Ryckman

August 21, 2026

Abstract

We propose that the recursive phase–transport structure developed for the twin-prime construction may have a corresponding organization in physical law. Before individual physical quantities can be assigned to the numerical states of that hierarchy, however, a more basic problem must be resolved: the physical system requires a well-defined quantity to serve as the normalized unit 1. This paper addresses that foundational problem.

The hydrogen Balmer series is used as the starting physical system because it provides a measured spectral hierarchy with a natural limiting boundary. The Balmer-limit wavelength is interpreted as one completed physical scale and normalized as a unit, while the remaining spectral lines are expressed as fractional positions relative to that completion. In this way, the dimensional Balmer scale is retained as a measurable length while simultaneously providing a dimensionless internal coordinate system.

From the completed Balmer length, a corresponding time coordinate is constructed using the light-crossing interval of one Balmer unit. Length and time may then both be represented in normalized Balmer coordinates, allowing the speed of light to appear as the completed propagation relation between one unit of length and one unit of time. General velocity and acceleration follow from the same ordinary kinematic relations after conversion into the new coordinates.

The purpose of this construction is not to replace standard physical equations, but to establish an independent normalized framework in which the proposed correspondence with the recursive phase–transport hierarchy can be tested. In particular, the present paper focuses only on the primitive physical unit, the construction of length and time coordinates, and the resulting descriptions of velocity and acceleration. The identification of gravitational, electromagnetic, particle, and higher recursive states is reserved for subsequent work. The resulting Balmer-normalized system therefore provides the proposed physical unit 1 from which the broader recursive hierarchy may be investigated.

1 Introduction: The Missing Physical Unit

The recursive phase–transport construction begins from a remarkably small numerical foundation. The states 2 and 3 establish a common six-unit structure, their corresponding transports combine into the first completed 5-state, and subsequent structures are generated by carrying completed states and their phase information forward. This raises a natural question if the same type of organization is to be investigated in physics: before asking what physical systems correspond to the states 2, 3, 5, or any later level, what physical quantity corresponds to the fundamental unit 1?

This is primarily a problem of calibration. Physical equations are normally written in units chosen to measure particular properties—meters, seconds, joules, kilograms, and so forth. Those units are extremely useful for measurement, but they do not by themselves identify a single physical scale from which the relationships between different quantities must be constructed. For the present framework, such a scale is required. A completed physical quantity must first be identified and treated as one unit before its internal fractions, transitions, and relationships to higher states can be compared with the recursive numerical construction.

Hydrogen provides a natural place to search for such a scale. Its spectral lines form an ordered and measurable system with a well-defined limiting boundary. In particular, the Balmer series approaches a finite limiting wavelength as its upper energy level increases without bound. Rather than treating this limit only as the endpoint of a spectroscopic series, we propose to use it as the basis of a physical normalization. The Balmer limit retains its ordinary dimensional value, but within the new coordinate system one completed Balmer scale is assigned the normalized value

$$1.$$

The purpose of this normalization is not merely to introduce another convenient unit of length. Once a fundamental length has been established, a corresponding time interval can be defined by the propagation of light across that length. Distance and time can then be expressed relative to the same physical calibration, allowing velocity to emerge from their relationship and acceleration to be constructed from the resulting velocity scale.

The first goal of this paper is therefore deliberately limited. We construct a Balmer-based unit system and show that the ordinary relations defining velocity and acceleration can be translated into it without altering their physical content. The numerical values used in conventional units may change, but the relationships represented by the equations remain unchanged.

This provides the first test required by the broader conjecture. If physical law and the recursive phase–transport hierarchy are different descriptions of the same underlying organization, then the correspondence should not begin by assigning familiar forces or particles to selected numerical positions. It should begin by establishing a common origin from which those positions can be defined. The Balmer unit developed here is proposed as that origin.

With this calibration in place, later work can investigate whether completed physical relations themselves occupy the reflected, phased, and recursively carried positions predicted by the numerical construction. The immediate task, however, is more fundamental: to construct the physical unit 1, and from it establish the first relationships between distance, time, velocity, and acceleration.

2 The Balmer Unit

For the Balmer series, the hydrogen spectrum may be written in the form

$$\frac{1}{\lambda_n} = R_\infty \left(\frac{1}{2^2} - \frac{1}{n^2} \right), \qquad n = 3, 4, 5, \ldots$$

where R_∞ is the Rydberg constant and λ_n is the wavelength associated with the transition from the level n to the Balmer boundary at $n = 2$.

As n increases without bound,

$$\frac{1}{n^2} \longrightarrow 0,$$

and the series approaches the limiting wavelength B, defined by

$$\frac{1}{B} = R_\infty \left(\frac{1}{4} \right).$$

Therefore,

$$\boxed{B = \frac{4}{R_\infty}.}$$

Using the standard value of the Rydberg constant gives approximately

$$\boxed{B \approx 3.6450682 \times 10^{-7} \text{ m}}$$

or

$$\boxed{B \approx 364.50682 \text{ nm.}}$$

We define this completed Balmer length as the fundamental length unit of the present coordinate system:

$$\boxed{1\,B = 1.}$$

The dimensional value has not been removed. Rather,

$$1\,B = 3.6450682 \times 10^{-7} \text{ m}$$

provides the conversion between the Balmer coordinate system and the metric system.

Any length L may therefore be represented in Balmer units by

$$\boxed{L_B = \frac{L}{B}.}$$

Thus the Balmer limit itself has the normalized coordinate

$$B_B = \frac{B}{B} = 1.$$

The same normalization may be applied directly to the spectral lines. Starting from

$$\frac{1}{\lambda_n} = R_\infty \left(\frac{1}{4} - \frac{1}{n^2} \right)$$

and using

$$\frac{1}{B} = \frac{R_\infty}{4},$$

we obtain

$$\boxed{\frac{B}{\lambda_n} = 1 - \left(\frac{2}{n} \right)^2.}$$

The Balmer series is therefore naturally expressed as a collection of fractional positions relative to the completed unit B.

This normalization also has an immediate energy interpretation. The energy carried by a photon of wavelength λ is

$$E = \frac{hc}{\lambda}.$$

The energy corresponding to the Balmer limit is therefore

$$E_B = \frac{hc}{B}.$$

Since

$$B = \frac{4}{R_\infty},$$

this may also be written as

$$\boxed{E_B = \frac{hcR_\infty}{4}.}$$

Numerically,

$$E_B \approx 3.4014 \text{ eV}.$$

If this completed Balmer energy is also normalized as one unit,

$$E_B = 1,$$

then every Balmer photon has the dimensionless energy coordinate

$$\frac{E_n}{E_B} = \frac{hc/\lambda_n}{hc/B} = \frac{B}{\lambda_n}.$$

Consequently,

$$\boxed{\frac{E_n}{E_B} = 1 - \left(\frac{2}{n}\right)^2.}$$

The same Balmer boundary therefore supplies both a dimensional length scale and its corresponding normalized energy scale. The single physical completion

$$\boxed{B = 1}$$

may be viewed either as one Balmer length or, through the photon relation, as one completed Balmer energy.

This provides the fundamental unit required for the remainder of the construction. The next step is to associate one Balmer length with its corresponding propagation time, producing the first completed relation between distance and time.

3 The Balmer Time Unit

The Balmer unit established in the previous section defines the completed length scale

$$B = \frac{4}{R_\infty},$$

with the approximate metric value

$$B \approx 3.6450682 \times 10^{-7} \text{ m}.$$

We now define the corresponding unit of time as the interval required for light to propagate across one Balmer length.

Using the measured speed of light,

$$c = 299792458 \ \mathrm{m\,s^{-1}},$$

the Balmer time unit is

$$\boxed{T_B = \frac{B}{c}.}$$

Substituting the metric value of B,

$$T_B = \frac{3.6450682 \times 10^{-7} \ \mathrm{m}}{299792458 \ \mathrm{m\,s^{-1}}},$$

which gives

$$\boxed{T_B \approx 1.215864 \times 10^{-15} \ \mathrm{s}.}$$

Thus the fundamental length and time conversions of the Balmer coordinate system are

$$\boxed{1\,B \approx 3.6450682 \times 10^{-7} \ \mathrm{m}}$$

and

$$\boxed{1\,T_B \approx 1.215864 \times 10^{-15} \ \mathrm{s}.}$$

Within the normalized Balmer coordinate system, these completed units are written

$$\boxed{1\,B = 1, \qquad 1\,T_B = 1.}$$

The relationship between them follows immediately from the definition of T_B. Since

$$T_B = \frac{B}{c},$$

rearranging gives

$$\boxed{c = \frac{B}{T_B}.}$$

Therefore one Balmer length traversed during one Balmer time is exactly one light-speed propagation interval:

$$\boxed{\frac{1\,B}{1\,T_B} = c.}$$

If velocity itself is expressed relative to this completed propagation scale, we may define the normalized Balmer velocity coordinate

$$v_B = \frac{v}{c}.$$

For light,

$$v = c,$$

and therefore

$$\boxed{v_B = 1.}$$

The normalized value 1 does not replace the dimensional value of the speed of light. The two representations are related directly:

$$\boxed{1 \text{ Balmer velocity unit} = 299792458 \text{ m s}^{-1}.}$$

We have therefore obtained a single matched coordinate system in which one unit of distance and one unit of time define one completed propagation relation:

$$\boxed{1\,B \quad \longleftrightarrow \quad 1\,T_B \quad \longrightarrow \quad v_B = 1.}$$

This gives the first kinematic relation constructed from the Balmer unit. We may now use the same coordinates to represent arbitrary velocity and, from velocity, acceleration.

4 Velocity and Acceleration in Balmer Coordinates

Let an arbitrary distance and time interval be expressed in Balmer coordinates as

$$x_B = \frac{x}{B}, \qquad t_B = \frac{t}{T_B}.$$

Therefore,

$$x = Bx_B, \qquad t = T_B t_B.$$

Beginning from the ordinary velocity relation,

$$v = \frac{\Delta x}{\Delta t},$$

substitution gives

$$v = \frac{B\,\Delta x_B}{T_B\,\Delta t_B}.$$

Since

$$\frac{B}{T_B} = c,$$

we obtain

$$v = c\frac{\Delta x_B}{\Delta t_B},$$

and therefore

$$\boxed{\frac{v}{c} = \frac{\Delta x_B}{\Delta t_B}.}$$

Thus the normalized Balmer velocity coordinate is

$$\boxed{v_B = \frac{v}{c} = \frac{\Delta x_B}{\Delta t_B}.}$$

The completed propagation state $v = c$ corresponds to

$$v_B = 1,$$

while arbitrary velocities are represented directly as fractions or multiples of this completed state.

Acceleration follows in the same coordinates. Beginning from

$$a = \frac{\Delta v}{\Delta t},$$

and using

$$v = cv_B, \qquad t = T_B t_B,$$

gives

$$a = \frac{c\,\Delta v_B}{T_B\,\Delta t_B}.$$

The natural Balmer acceleration unit is therefore

$$\boxed{A_B = \frac{c}{T_B}.}$$

Since

$$T_B = \frac{B}{c},$$

this becomes

$$\boxed{A_B = \frac{c^2}{B}.}$$

Numerically,

$$A_B \approx 2.466 \times 10^{23}\ \mathrm{m\,s^{-2}}.$$

Defining normalized acceleration by

$$a_B = \frac{a}{A_B},$$

we obtain

$$\boxed{a_B = \frac{aB}{c^2}.}$$

Equivalently,

$$\boxed{a_B = \frac{\Delta v_B}{\Delta t_B}.}$$

The ordinary kinematic relations are therefore preserved exactly under the Balmer normalization:

$$\boxed{v_B = \frac{\Delta x_B}{\Delta t_B}, \qquad a_B = \frac{\Delta v_B}{\Delta t_B}.}$$

The Balmer coordinate system has now produced consistent units of distance, time, velocity, and acceleration from the single completed length scale B. In particular, the acceleration scale

$$\boxed{A_B = \frac{c^2}{B}}$$

provides the quantity required for the next comparison with gravitational acceleration.

5 The First Recursive Kinematic Structure

The Balmer normalization has now produced the sequence

$$B \longrightarrow T_B \longrightarrow c \longrightarrow A_B,$$

with

$$B = 1, \qquad T_B = 1, \qquad c = \frac{B}{T_B}, \qquad A_B = \frac{c^2}{B}.$$

This gives the first simple example of the recursive structure proposed in this framework. Distance and time are the two primitive coordinates required to define velocity. Thus, although the completed Balmer velocity has the normalized numerical value

$$c = 1,$$

its structural relation contains two components:

$$\boxed{\text{distance} + \text{time} \longrightarrow 2.}$$

Here the addition denotes the number of primitive coordinate types participating in the relation; it does not replace the ordinary velocity equation

$$v = \frac{\Delta x}{\Delta t}.$$

The completed velocity relation is then carried directly into the next kinematic quantity. From the preceding section,

$$A_B = \frac{c}{T_B}.$$

Using

$$T_B = \frac{B}{c},$$

gives

$$\boxed{A_B = \frac{c^2}{B}.}$$

Since the Balmer length is the normalized unit

$$B = 1,$$

the acceleration scale is governed by the squared propagation relation,

$$A_B \sim c^2.$$

Under the proposed structural counting, the two-component velocity state is therefore lifted through its square:

$$\boxed{2 \longrightarrow 2^2 = 4.}$$

This is the first physical example of a completed relation becoming an input to a higher relation. The numerical value of the completed unit remains normalized, while its structural order increases.

The resulting progression may therefore be summarized as

$$\boxed{1 \longrightarrow 2 \longrightarrow 4,}$$

where 1 represents the completed Balmer unit, 2 represents the distance–time relation defining propagation, and 4 represents the squared propagation structure appearing naturally in acceleration.

The next question is whether this acceleration structure connects directly to gravity. The simple pendulum provides an immediate test, because its equation relates exactly the three quantities already constructed here: distance, time, and acceleration.

6 The Gravitational Boundary and the Pendulum

The simple pendulum provides a direct connection between the Balmer kinematic coordinates and gravitational acceleration. Its period is

$$T = 2\pi\sqrt{\frac{L}{g}},$$

or equivalently,

$$\boxed{g = \frac{4\pi^2 L}{T^2}.}$$

All three quantities appearing in this relation have already been defined within the Balmer coordinate system. We write

$$L = BL_B,$$

$$T = T_B t_B,$$

and

$$g = A_B g_B,$$

where

$$A_B = \frac{c^2}{B}.$$

Substitution into the pendulum equation gives

$$A_B g_B = \frac{4\pi^2 B L_B}{T_B^2 t_B^2}.$$

Since

$$T_B = \frac{B}{c},$$

we have

$$\frac{B}{T_B^2} = \frac{c^2}{B} = A_B.$$

The acceleration scale therefore cancels identically, leaving

$$\boxed{g_B = \frac{4\pi^2 L_B}{t_B^2}.}$$

Thus the pendulum relation has exactly the same mathematical form in Balmer coordinates as it does in ordinary dimensional coordinates:

$$\boxed{g = \frac{4\pi^2 L}{T^2} \qquad \longleftrightarrow \qquad g_B = \frac{4\pi^2 L_B}{t_B^2}.}$$

No modification of the gravitational relation is required. The Balmer system changes only the coordinate scale in which the quantities are measured.

As a direct numerical example, consider the standard gravitational acceleration

$$g_0 = 9.80665 \text{ m s}^{-2}.$$

Using

$$A_B = \frac{c^2}{B} \approx 2.46567 \times 10^{23} \text{ m s}^{-2},$$

the same acceleration in Balmer coordinates is

$$\boxed{g_{0,B} = \frac{g_0}{A_B} = \frac{g_0 B}{c^2} \approx 3.97727 \times 10^{-23}.}$$

Now consider a pendulum having a full period of

$$T = 2 \text{ s}.$$

Its Balmer time coordinate is

$$t_B = \frac{2}{T_B} \approx 1.64492 \times 10^{15}.$$

Substitution into the Balmer pendulum equation gives

$$L_B = \frac{g_{0,B} t_B^2}{4\pi^2} \approx 2.72593 \times 10^6.$$

Converting this result back into meters,

$$L = BL_B,$$

gives

$$\boxed{L \approx 0.993621 \text{ m.}}$$

This is precisely the value obtained directly from the ordinary pendulum equation,

$$L = \frac{g_0 T^2}{4\pi^2}.$$

The Balmer normalization therefore reproduces the same measured gravitational relationship without changing the underlying physics. Length, time, velocity, acceleration, and now gravitational acceleration may all be represented consistently within the same coordinate system.

The pendulum relation consequently provides the first direct bridge from the Balmer kinematic construction into gravity. A subsequent treatment may begin from this equality and extend the same framework into force, mass, gravitational coupling, and the higher physical states.

7 Discussion

The construction developed here is intentionally elementary. Beginning from the Balmer limit, we defined a common physical unit and showed that distance, time, velocity, acceleration, and gravitational acceleration may all be expressed in the resulting coordinates without modifying the equations that already describe them.

The broader conjecture is substantially stronger. We propose that the recursive phase–transport structure previously identified in the twin-prime construction is not limited to number theory, but provides an organizing framework into which physical relationships may also be mapped. Under this interpretation, familiar equations are not replaced. Rather, each equation occupies a structural position in a larger recursive system of completed states, reflected relations, phase offsets, mediated states, and subsequent closures.

The present paper establishes only the common origin required for that comparison. Once the physical unit 1 has been fixed, the remaining problem becomes one of determining where the known equations and measured relationships of physics occupy the recursive hierarchy.

Our working conjecture is that this mapping extends throughout physics and ultimately provides a unified description of apparently separate physical domains. If this is correct, the problem of unification may be considerably simpler than constructing an entirely new collection of physical laws: the existing laws may already be the local expressions of a single recursive structure.

The next immediate target is the transition between gravity and electromagnetism. In the phase–transport construction, the primitive states 2 and 3 share the common completion

$$2(3) = 3(2) = 6.$$

The corresponding physical hypothesis is that gravitational and electromagnetic descriptions occupy neighboring states whose interaction is completed through an analogous six-state structure. The fine-structure constant is therefore a natural quantity to investigate as a possible phase offset, mediated relation, or transition coordinate between those states.

This interpretation will be developed in subsequent work rather than assumed here. The same approach may then be extended to mass, momentum, charge, particle structure, atomic organization, and ultimately the periodic table.

The scope of this program is much larger than the work of any individual author. The scientific community collectively possesses far greater experimental, mathematical, and theoretical expertise than can be brought to the problem here. The purpose of presenting the framework openly is therefore not only to continue developing these correspondences, but to make the structure available for independent testing, criticism, extension, and rapid exploration.

Until that broader investigation occurs, we will continue to develop the map incrementally, beginning with the simplest physical relationships and moving outward through the recursive hierarchy.

8 Conclusion

This paper began with a single question: if the recursive phase–transport hierarchy has a physical correspondence, what physical quantity should be identified with the fundamental unit 1?

The hydrogen Balmer limit provides a natural answer. Defining the completed Balmer length as one unit gives

$$B = \frac{4}{R_\infty},$$

with a corresponding time unit

$$T_B = \frac{B}{c}.$$

These immediately produce the normalized propagation relation

$$c = \frac{B}{T_B},$$

and the associated acceleration scale

$$A_B = \frac{c^2}{B}.$$

The ordinary equations for velocity, acceleration, and the simple pendulum retain their form under this normalization. The Balmer system therefore supplies a common coordinate origin without requiring any modification of the physical relationships already in use.

This establishes the foundation required for the broader phase–transport conjecture. Our proposed direction is that the known equations of physics occupy positions within the same recursive hierarchy previously identified in the numerical construction, with completed relations carried forward into higher states and reflected or mediated relationships connecting neighboring domains.

The next step is to investigate gravity and electromagnetism within this structure, with particular attention to the fine-structure constant as a possible phase relation between the first neighboring physical states. From there the same method may be extended toward mass, charge, particle structure, and atomic organization.

If the correspondence continues as proposed, the resulting framework would provide a candidate route toward a Grand Unified Theory in which the apparent diversity of physical equations arises from one recursive underlying structure. The immediate task is now straightforward: continue the mapping, test each correspondence independently, and determine how far the common structure extends.

Acknowledgments and Intellectual Influences

The development of this work was influenced not only by formal scientific literature, but also by many years of public mathematics and science communication.

I would particularly like to acknowledge Grant Sanderson and *3Blue1Brown*, whose geometric approach to mathematics has strongly influenced the way I visualize mathematical structure; Brady Haran and the many mathematicians appearing on *Numberphile*; and Burkard Polster's *Mathologer*, whose explorations of mathematical patterns helped encourage the kind of structural experimentation used throughout this work.

I would also like to acknowledge the presenters, writers, researchers, producers, and entire team behind the Canadian science program *@discovery.ca*. Watching the program during my early exposure to science contributed significantly to the curiosity and interdisciplinary viewpoint that eventually led to the present work.

Finally, I would especially like to acknowledge Michio Kaku. His ability to communicate large questions in theoretical physics—particularly the search for unification and the possibility that apparently different physical phenomena may emerge from a deeper common structure—was an important influence on my decision to pursue these questions independently.

These individuals and programs are not cited as sources for the mathematical derivations presented in this paper. Rather, they are acknowledged as intellectual and educational influences that helped shape the way the author approached the problem.

References

[1] Patrick Ryckman, *The Recursive Phase–Transport Map: Forward Generation of Consecutive Twin-Prime Centers*, 2026.

[2] Johann Jakob Balmer, "Notiz über die Spectrallinien des Wasserstoffs," *Annalen der Physik und Chemie*, vol. 261, no. 5, pp. 80–87, 1885.

[3] Johannes Rydberg, works on the spectral-series relation now represented by the Rydberg constant and Rydberg formula.

[4] National Institute of Standards and Technology, *CODATA Recommended Values of the Fundamental Physical Constants*.

[5] Grant Sanderson, *3Blue1Brown*, mathematics education and visualization project, `https://www.3blue1brown.com/`.

[6] Brady Haran and contributors, *Numberphile*, mathematics communication project, `https://www.numberphile.com/`.

[7] Burkard Polster, *Mathologer*, mathematics education and visualization project, `https://www.youtube.com/@Mathologer`.

[8] *@discovery.ca*, Canadian television science program, Discovery Channel Canada.

[9] Michio Kaku, popular and scholarly works on theoretical physics, unification, higher-dimensional physics, and science communication.